TRIGONOMETRIC IDENTITIES

LESSON 14 UNIT 01

OBJECTIVES

STUDENTS WILL BE ABLE TO:

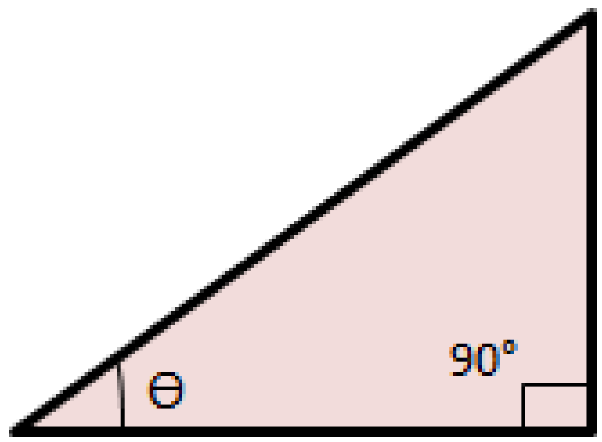
write any basic trigonometric identity given a right-angled triangle

KEY VOCABULARY:

- Right-angled triangle
- Hypotenuse, opposite, adjacent and angle Ɵ
- Sine, Cosine and Tangent
- Cotangent, Cosecant and Secant
- Reciprocal, Quotient and Pythagorean identities

TRIGONOMETRIC IDENTITIES

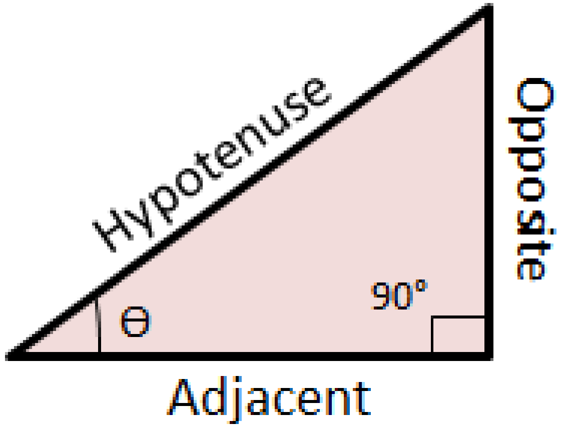
A **Right-angled triangle** is a triangle which has one of its angles equal to 90 degrees. The remaining two angles always have a sum equal to 90 degrees.



TRIGONOMETRIC IDENTITIES

There are properties associated with a right-angled triangle.

- A **hypotenuse** is the line segment opposite to the right-angle.
- An **opposite** is the line segment opposite to the angle Θ.
- An **adjacent** is the line segment next to the angle Θ.

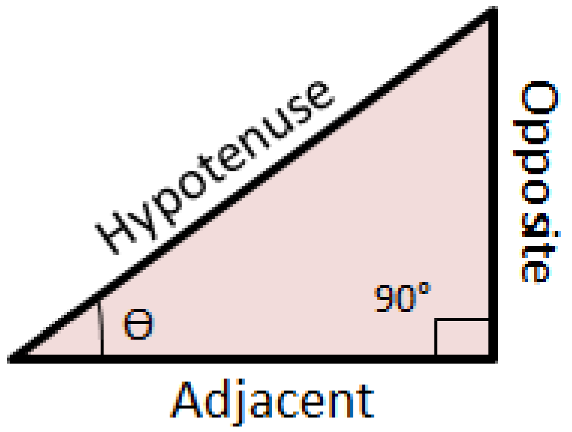


TRIGONOMETRIC IDENTITIES

There are six basic trigonometric identities that can be written given a right-angled triangle.

Sine:

$$\sin(\theta) = \frac{opposite}{hypotenuse}$$



Cosine:

$$\cos(\theta) = \frac{adjacent}{hypotenuse}$$

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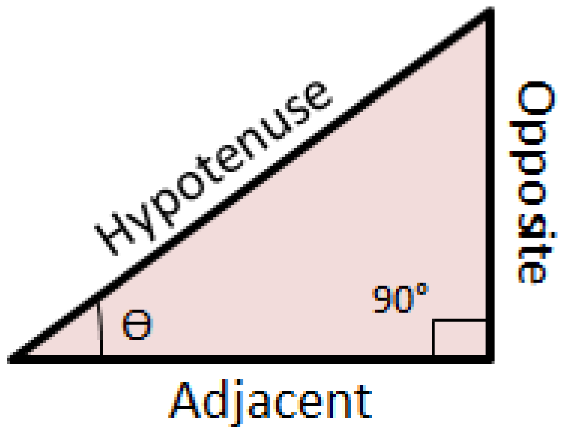
Tangent:

$$tan(\theta) = \frac{opposite}{adjacent}$$

Cotangent:

$$cot(\theta) = \frac{adjacent}{opposite}$$



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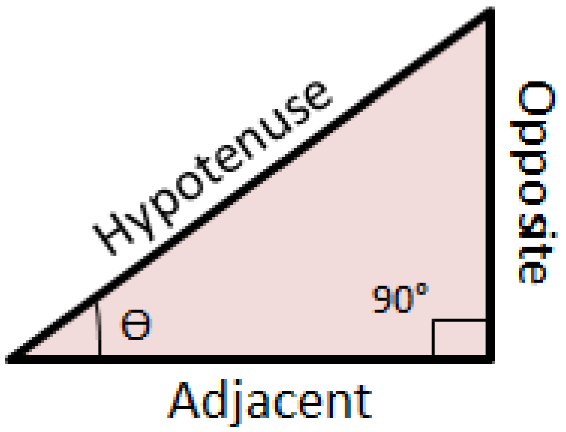
Cosecant:

$$cosec(\theta) = \frac{hypotenuse}{opposite}$$



Secant:

$$sec(\theta) = \frac{hypotenuse}{adjacent}$$

TRIGONOMETRIC IDENTITIES

Reciprocal identities relate these six identities such that one identity is the reciprocal of its co-identity.

Sine and Cosecant:

$$sin(\theta) = \frac{opposite}{hypotenuse} = \frac{1}{\frac{hypotenuse}{opposite}} = \frac{1}{cosec(\theta)}$$

$$\boldsymbol{sin(\theta) = \frac{1}{cosec(\theta)}} \quad ; \quad \boldsymbol{cosec(\theta) = \frac{1}{sin(\theta)}}$$

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Cosine and Secant:

$$cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{1}{\frac{hypotenuse}{adjacent}} = \frac{1}{sec(\theta)}$$

$$\boldsymbol{cos(\theta) = \frac{1}{sec(\theta)}} \quad ; \quad \boldsymbol{sec(\theta) = \frac{1}{cos(\theta)}}$$

Tangent and Cotangent:

$$tan(\theta) = \frac{opposite}{adjacent} = \frac{1}{\frac{adjacent}{opposite}} = \frac{1}{cot(\theta)}$$

$$\boldsymbol{tan(\theta) = \frac{1}{cot(\theta)}} \quad ; \quad \boldsymbol{cot(\theta) = \frac{1}{tan(\theta)}}$$

TRIGONOMETRIC IDENTITIES

Quotient identities relate the sine and cosine with tangent and cotangent of an angle in a right-angled triangle.

Tangent:

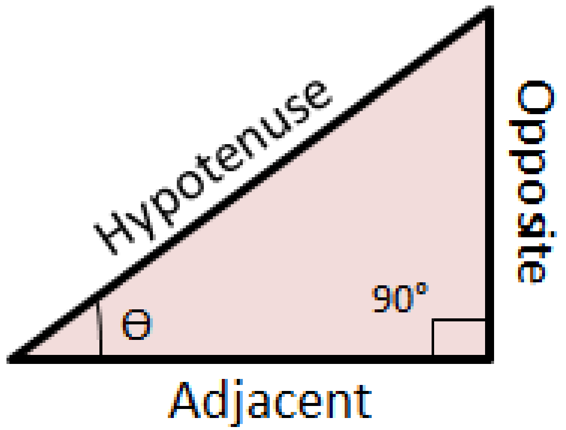
$$tan(\theta) = \frac{opposite}{adjacent} = \frac{\frac{opposite}{hypotenuse}}{\frac{adjacent}{hypotenuse}} = \frac{sin(\theta)}{cos(\theta)}$$

Cotangent:

$$cot(\theta) = \frac{adjacent}{opposite} = \frac{\frac{adjacent}{hypotenuse}}{\frac{opposite}{hypotenuse}} = \frac{cos(\theta)}{sin(\theta)}$$

TRIGONOMETRIC IDENTITIES

Pythagorean identities are written using the Pythagorean theorem for right-angled triangles.



Pythagorean theorem

$$(Hypotenuse)^2 = (Opposite)^2 + (Adjacent)^2$$

Dividing both sides by $(Hypotenuse)^2$, we get

$$1 = \frac{(Opposite)^2}{(Hypotenuse)^2} + \frac{(Adjacent)^2}{(Hypotenuse)^2} \rightarrow 1 = \left(\frac{Opposite}{Hypotenuse}\right)^2 + \left(\frac{Adjacent}{Hypotenuse}\right)^2$$

$$\mathbf{1 = sin^2(\theta) + cos^2(\theta)}$$

TRIGONOMETRIC IDENTITIES

Similarly, other two Pythagorean identities can be written as:

$$\mathbf{1 + \boldsymbol{tan^2(\theta)} = \boldsymbol{sec^2(\theta)}}$$

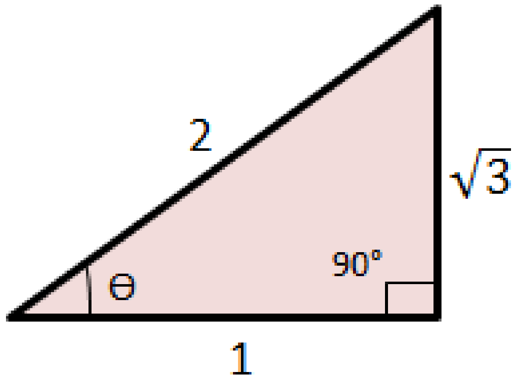
$$\mathbf{1 + \boldsymbol{cot^2(\theta)} = \boldsymbol{cosec^2(\theta)}}$$

TRIGONOMETRIC IDENTITIES

PROBLEM 1:

Consider the right-angled triangle in the figure and write the six basic trigonometric identities.

Here, $opposite = \sqrt{3}$; $adjacent = 1$; $hypotenuse = 2$



$$\sin(\theta) = \frac{\sqrt{3}}{2} \quad ; \quad \cos(\theta) = \frac{1}{2}$$

$$\tan(\theta) = \sqrt{3} \quad ; \quad \cot(\theta) = \frac{1}{\sqrt{3}}$$

$$\mathrm{cosec}(\theta) = \frac{2}{\sqrt{3}}; \quad \sec(\theta) = 2$$

TRIGONOMETRIC IDENTITIES

PROBLEM 2:

Find $\cos(\theta)$, $\cot(\theta)$, $\operatorname{cosec}(\theta)$ and $\sec(\theta)$ if:

$\sin(\theta) = \frac{1}{2}$ and $\tan(\theta) = 1$

We can use the quotient identity and reciprocal identities to find these identities.

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \text{ or } \cos(\theta) = \frac{\sin(\theta)}{\tan(\theta)} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{1}{1} = 1 \;\; ; \operatorname{cosec}(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{\frac{1}{2}} = 2$$

And, $\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{\frac{1}{2}} = 2$