Permutations and Combinations

Unit 6 Lesson 7
PERMUTATIONS AND COMBINATIONS

Students will be able to:
Understand the concept of permutations and combinations and their usage.

Key Vocabulary

• Principle of Counting
• Permutation
• Combination
• Factorial
What is the Fundamental Principle of Counting?

Suppose that a task 1 can be done in \( n_1 \) ways, and a task 2 can be done in \( n_2 \) ways, ..., and a task \( k \) can be done in \( n_k \) ways, then all of these tasks can be done in:

\[
n_1 \times n_2 \times ... \times n_k \text{ ways.}
\]
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PROBLEM: 1

Alan bought 7 dress shirts, 5 pants and 3 shoes from a shopping mall. How many different combinations of pants, dress shirts and shoes can he make from his shopping?

Here, there are 7 dress shirts, so they can be selected in 7 different ways, so \( n_1 = 7 \).

There are 5 pants, so they can be selected in 5 different ways, so \( n_2 = 5 \).

There are 3 shoes, so they can be selected in 3 different ways, so \( n_3 = 3 \). Now all the three things can be selected in:

\[
7 \times 5 \times 3 = 105 \text{ ways}
\]

Hence Alan can make 105 combinations.
What is Permutation?
A permutation is the choice of \( r \) things from a set of \( n \) things without replacement and where the arrangement (order) matters.

Mathematically,

\[
\binom{n}{r}P = \frac{n!}{(n-r)!}
\]

Where,

\( n! = n \times (n - 1) \times (n - 2) \times \cdots \times 3 \times 2 \times 1 \) (Termed as the \( n \)-Factorial)

Also, \( 0! = 1 \).
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PROBLEM: 2

Find the following:

a) \( \binom{n}{0} \)

b) \( \binom{5}{2} \)

a) \[ \binom{n}{0} = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1 \]

b) \[ \binom{5}{2} = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 5 \times 4 = 20 \]
What is Combination?
A combination is the choice of \( r \) things from a set of \( n \) things without replacement and where the arrangement (order) does not matter.

Mathematically,

\[
{n \choose r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}
\]

Where,

\[
n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1 \quad \text{(Termed as the n-factorial)}
\]
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PROBLEM: 3

Find the following:

a) \( \binom{n}{0} \)

b) \( \binom{5}{2} \)

a) \( \binom{n}{0} = \frac{n!}{0!(n-0)!} = \frac{n!}{1 \times n!} = 1 \)

b) \( \binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5!}{2! \times 3!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times (3 \times 2 \times 1)} = \frac{5 \times 4}{2} = 10 \)

Note that \( \binom{n}{0} = \binom{n}{0} P = 1 \)