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# Permutations and Combinations

Unit 6 Lesson 7

# PERMUTATIONS AND COMBINATIONS

Students will be able to:

Understand the concept of permutations and combinations and their usage.

## Key Vocabulary

- **Principle of Counting**
- **Permutation**
- **Combination**
- **Factorial**

# PERMUTATIONS AND COMBINATIONS

## What is the Fundamental Principle of Counting?

Suppose that a task **1** can be done in  $n_1$  ways, and a task **2** can be done in  $n_2$  ways, ..., and a task  **$k$**  can be done in  $n_k$  ways, then all of these tasks can be done in:

$$n_1 \times n_2 \times \dots \times n_k \text{ ways.}$$

# PERMUTATIONS AND COMBINATIONS

## PROBLEM: 1

Alan bought 7 dress shirts, 5 pants and 3 shoes from a shopping mall. How many different combinations of pants, dress shirts and shoes can he make from his shopping?

Here, there are 7 dress shirts, so they can be selected in 7 different ways, so  $n_1 = 7$ .

There are 5 pants, so they can be selected in 5 different ways, so  $n_2 = 5$ .

There are 3 shoes, so they can be selected in 3 different ways, so  $n_3 = 3$ . Now all the three things can be selected in:

$$7 \times 5 \times 3 = 105 \text{ ways}$$

Hence Alan can make 105 combinations.

# PERMUTATIONS AND COMBINATIONS

## What is Permutation?

A permutation is the choice of  $r$  things from a set of  $n$  things without replacement and where the arrangement (order) matters.

Mathematically,

$${}_r P_n = \frac{n!}{(n-r)!}$$

Where,

$n! = n \times (n - 1) \times (n - 2) \times \dots \times 3 \times 2 \times 1$  (Termed as the **n-Factorial**)

Also,  $0! = 1$ .

# PERMUTATIONS AND COMBINATIONS

## PROBLEM: 2

Find the following:

a)  ${}_0P^n$

b)  ${}_2P^5$

$$\text{a) } {}_0P^n = \frac{n!}{(n-0)!} = \frac{n!}{n!} = \mathbf{1}$$

$$\text{b) } {}_2P^5 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = \mathbf{5 \times 4 = 20}$$

# PERMUTATIONS AND COMBINATIONS

## What is Combination?

A combination is the choice of  $r$  things from a set of  $n$  things without replacement and where the arrangement (order) does not matter.

Mathematically,

$${}^n C_r = \binom{n}{r} = \frac{n!}{n!(n-r)!}$$

Where,

$$n! = n \times (n - 1) \times (n - 2) \times \cdots \times 3 \times 2 \times 1 \quad (\text{Termed as the } \mathbf{n\text{-factorial}})$$

# PERMUTATIONS AND COMBINATIONS

## PROBLEM: 3

Find the following:

a)  ${}^n_0C$

b)  ${}^5_2C$

$$\text{a) } {}^n_0C = \frac{n!}{0!(n-0)!} = \frac{n!}{1 \times n!} = \mathbf{1}$$

$$\text{b) } {}^5_2C = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{(2 \times 1) \times (3 \times 2 \times 1)} = \frac{5 \times 4}{2} = \mathbf{10}$$

Note that  ${}^n_0C = {}^n_0P = \mathbf{1}$