Probability of Multiple Events
Unit 9 Lesson 7

## PROBABILITY OF MULTIPLE EVENTS

## Students will be able to:

Understand the concept of probability and find the probability of multiple events.

## Key Vocabulary

- Probability
- Mutually Exclusive events
- Probability of multiple events
- Independent and Dependent events
- Probability of Independent and Dependent events

What is the probability of an event?
The probability of an event $\mathbf{A}$ is the number of ways event $\mathbf{A}$ can occur ( $\boldsymbol{k}$ ways) divided by the total number of possible outcomes $\mathbf{N}$.

$$
\boldsymbol{P}(\boldsymbol{A})=\frac{\text { number of ways event } \boldsymbol{A} \text { can occur }}{\text { total number of outcomes }}
$$

$$
P(A)=\frac{k}{N}
$$

## Multiple Events and Mutually Exclusive Events

Multiple events refer to the occurrence of more than one events that can be related to each other in some way or not related at all.

Mutually Exclusive events are the events that cannot happen at the same time.

For example, occurrence of Heads and Tails when flipping a coin is an example of mutually exclusive events.

## Probability of Multiple Events

If there are two mutually exclusive events $\mathbf{A}$ and $\mathbf{B}$ such that:
Probability of event $\mathrm{A}=P(A)=\frac{n(A)}{N}$
Probability of event $\mathrm{B}=P(B)=\frac{n(B)}{N}$

Then,

- The probability that both the events occur is

$$
P(A \cap B)=P(A) \cdot P(B)
$$

- The probability that either of the two events occur is

$$
P(A \cup B)=P(A)+P(B)-P(A \cap B)
$$

## Example: 1

Consider a dice is rolled twice and the two events under consideration are:
$A=a$ dice is rolled and an odd number occurs
$B=a$ dice is rolled and an even number occurs

1. Then the probability that both events occur is $\boldsymbol{P}(\boldsymbol{A} \cap \boldsymbol{B})=\boldsymbol{P}(\boldsymbol{A}) \cdot \boldsymbol{P}(\boldsymbol{B})$

$$
P(A)=\frac{3}{6}=\frac{1}{2} \quad ; \quad P(B)=\frac{3}{6}=\frac{1}{2}
$$

$$
P(A \cap B)=\frac{1}{2} \times \frac{1}{2}=\frac{1}{4}
$$

2. Then the probability that either of these events occur is $\boldsymbol{P}(\boldsymbol{A} \cup \boldsymbol{B})=\boldsymbol{P}(\boldsymbol{A})+$ $P(B)-P(A \cap B)$

$$
P(A \cup B)=\frac{1}{2}+\frac{1}{2}-\frac{1}{4}=\frac{3}{4}
$$

## Independent Events and Dependent Events

- Two events are said to be independent if the result of second event is not affected by the result of first event.
For example, the flipping of a coin multiple times is an independent event since flipping a coin is an isolated event.
- Two events are said to be dependent if the result of the second event is affected by the result of first event.
For example, if multiple colored balls are drawn from a pool of colored ball without replacement, the each withdrawal results in the decrease of the total number of balls. So these are dependent events.


## Probability of Independent Events

If there are two independent events $\mathbf{A}$ and $\mathbf{B}$ with probabilities $\boldsymbol{P}(\boldsymbol{A})$ and $\boldsymbol{P}(\boldsymbol{B})$, then the probability of occurrence of both events is:

$$
\boldsymbol{P}(\boldsymbol{A} \text { and } \boldsymbol{B})=\boldsymbol{P}(\boldsymbol{A}) \cdot \boldsymbol{P}(\boldsymbol{B})
$$

## Probability of dependent Events

If there are two dependent events $\mathbf{A}$ and $\mathbf{B}$ with probabilities $\boldsymbol{P}(\boldsymbol{A})$ and $\boldsymbol{P}(\boldsymbol{B})$, and $\mathbf{A}$ occurs first, then the probability of occurrence of both events is:

$$
\boldsymbol{P}(\boldsymbol{A} \text { and } \boldsymbol{B})=\boldsymbol{P}(\boldsymbol{A}) \cdot \boldsymbol{P}(\boldsymbol{B}, \text { once } \boldsymbol{A} \text { has occurred })
$$

## Example: 2

A dice is rolled and a coin is flipped. What is the probability that 6 comes on the dice and Heads comes on the coin?
$P(A)=P(6$ comes on the dice $)=\frac{1}{6}$

$$
P(B)=P(\text { Heads comes on the coin })=\frac{1}{2}
$$

Then the probability that 6 comes on the dice and Heads comes on the coin is:

$$
P(A \cap B)=P(A) \cdot P(B)=\frac{1}{6} \times \frac{1}{2}=\frac{1}{12}
$$

## Example: 3

There are 3 yellow balls, 4 green balls and 5 red balls in a basket. A ball is taken out at random and not replaced. Then another ball is taken out. What is the probability that the first ball is yellow and the second ball is green?

$$
\begin{aligned}
& P(A)=P(\text { yellow ball })=\frac{3}{12}=\frac{1}{4} \\
& P(B, \text { after } A \text { occurred })=P(\text { green ball, after yellow ball taken out })=\frac{4}{11}
\end{aligned}
$$

Then the probability that the first ball is yellow and the second ball is green is:

$$
P(\boldsymbol{A} \text { and } \boldsymbol{B})=P(\boldsymbol{A}) \cdot P(\boldsymbol{B}, \text { after } \boldsymbol{A})=\frac{1}{4} \times \frac{4}{11}=\frac{\mathbf{1}}{\mathbf{1 1}}
$$

