Name: ____ Period: __________ Date: _______________

AREA UNDER A CURVE Assignment

On graph paper graph and find the area between each curve and the x-axis for the given interval and subinterval using left sum, right sum, and middle sum.

1. $y = 2x^2$ from $x = -1$ to $x = 1, \mathrm{n} = 4$

2. $y = x^3$ from $x = 1$ to $x = 2, \mathrm{n} = 4$

3. $y = 3x + 1$ from $x = 0$ to $x = 1, \mathrm{n} = 4$

4. $y = x$ from $x = 1$ to $x = 4, \mathrm{n} = 6$

5. $y = \frac{x^3}{5}$ from $x = 1$ to $x = 3, \mathrm{n} = 8$

6. $y = \frac{x}{5}$ from $x = 1$ to $x = 3, \mathrm{n} = 4$

7. $y = 2x$ from $x = 0$ to $x = 2, \mathrm{n} = 4$

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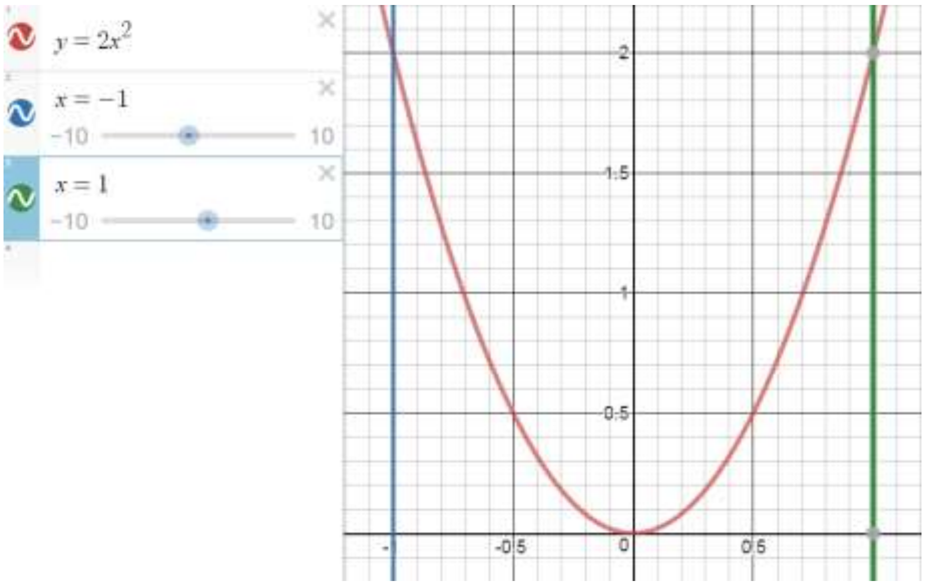
AREA UNDER A CURVE Assignment

ANSWER

Graph and find the area between each curve and the x-axis for the given interval and subinterval using left sum, right sum, and middle sum.

1. $y = 2x^2$ from $x = -1$ to $x = 1, n = 4$



$$\Delta x = \frac{b-a}{n}$$
$$\Delta x = \frac{1-(-1)}{4}$$
$$\Delta x = \frac{2}{4}$$
$$\Delta x = \frac{1}{2}$$

A. **LEFT SUM**

$$x_{n-1} = a + (n-1)\cdot \Delta x = -1 + (n-1)\left(\frac{1}{2}\right)$$
$$x_{n-1} = -1 + \frac{n-1}{2} = -\frac{2}{2} + \frac{n-1}{2} = \frac{n-2-1}{2}$$
$$x_{n-1} = \frac{n-3}{2}$$

$$f(x_{n-1}) = f\left(\frac{n-3}{2}\right) = 2\left(\frac{n-3}{2}\right)^2$$
$$f(x_{n-1}) = 2\left(\frac{n-3}{2}\right)^2$$

$$A_n(left\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f(x_{n-1})$$
$$A_4(left\ sum) = \sum_{n=1}^{4}\left(\frac{1}{2}\right)\cdot 2\left(\frac{n-3}{2}\right)^2$$
$$A_4(left\ sum) = \sum_{n=1}^{4}\left(\frac{n-3}{2}\right)^2$$
$$A_4(left\ sum) = \frac{3}{2} = 1.5$$

B. **RIGHT SUM**

$$x_n = a + n\cdot \Delta x = -1 + n\left(\frac{1}{2}\right) = -\frac{2}{2} + \frac{n}{2}$$
$$x_n = \frac{n-2}{2}$$

$$f(x_n) = f\left(\frac{n-2}{2}\right) = 2\left(\frac{n-2}{2}\right)^2$$
$$f(x_n) = 2\left(\frac{n-2}{2}\right)^2$$

$$A_n(\text{right}\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f(x_n)$$
$$A_4(\text{right}\ sum) = \sum_{n=1}^{4}\left(\frac{1}{2}\right)\cdot 2\left(\frac{n-2}{2}\right)^2$$
$$A_4(\text{right}\ sum) = \sum_{n=1}^{4}\left(\frac{n-2}{2}\right)^2$$
$$A_4(\text{right}\ sum) = \frac{3}{2} = 1.5$$

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AREA UNDER A CURVE Assignment

C. **MIDDLE SUM**

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{n-3}{2} + \frac{n-2}{2}}{2} = \frac{\frac{n-3+n-2}{2}}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{2n-5}{2}}{2} = \frac{2n-5}{2} \cdot \frac{1}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{2n-5}{4}$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = f\left(\frac{2n-5}{4}\right) = 2\left(\frac{2n-5}{4}\right)^2$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = 2\left(\frac{2n-5}{4}\right)^2$$

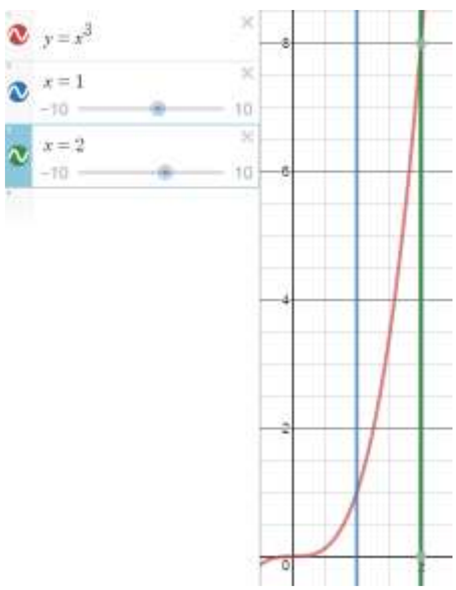
$$A_n(middle\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_n + x_{n-1}}{2}\right)$$

$$A_4(middle\ sum) = \sum_{n=1}^{4}\left(\frac{1}{2}\right) \cdot 2\left(\frac{2n-5}{4}\right)^2$$

$$A_4(middle\ sum) = \sum_{n=1}^{4}\left(\frac{2n-5}{4}\right)^2$$

$$A_4(middle\ sum) = \frac{5}{4} = 1.25$$

2. $y = x^3$ from $x = 1$ to $x = 2, n = 4$



$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{2-1}{4}$$

$$\Delta x = \frac{1}{4}$$

A. **LEFT SUM**

$$x_{n-1} = a + (n-1) \cdot \Delta x = 1 + (n-1)\left(\frac{1}{4}\right)$$

$$x_{n-1} = 1 + \frac{n-1}{4} = \frac{4}{4} + \frac{n-1}{4} = \frac{n-1+4}{4}$$

$$x_{n-1} = \frac{n+3}{4}$$

$$f(x_{n-1}) = f\left(\frac{n+3}{4}\right) = \left(\frac{n+3}{4}\right)^3$$

$$f(x_{n-1}) = \left(\frac{n+3}{4}\right)^3$$

$$A_n(left\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right) \cdot f(x_{n-1})$$

$$A_4(left\ sum) = \sum_{n=1}^{4}\left(\frac{1}{4}\right) \cdot \left(\frac{n+3}{4}\right)^3$$

$$A_4(left\ sum) = \frac{1}{4}\sum_{n=1}^{4}\left(\frac{n+3}{4}\right)^3 = \frac{1}{4}\left(\frac{187}{16}\right)$$

$$A_4(left\ sum) = \frac{187}{64} = 2.921875$$

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AREA UNDER A CURVE Assignment

B. **RIGHT SUM**

$$x_n = a + n \cdot \Delta x = 1 + n\left(\frac{1}{4}\right) = \frac{4}{4} + \frac{n}{4}$$

$$x_n = \frac{n+4}{4}$$

$$f(x_n) = f\left(\frac{n+4}{4}\right) = \left(\frac{n+4}{4}\right)^3$$

$$f(x_n) = \left(\frac{n+4}{4}\right)^3$$

$$A_n(\text{right } sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right) \cdot f(x_n)$$

$$A_4(\text{right } sum) = \sum_{n=1}^{4}\left(\frac{1}{4}\right) \cdot \left(\frac{n+4}{4}\right)^3$$

$$A_4(\text{right } sum) = \frac{1}{4}\sum_{n=1}^{4}\left(\frac{n+4}{4}\right)^3 = \frac{1}{4}\left(\frac{299}{16}\right)$$

$$A_4(\text{right } sum) = \frac{299}{64} = 4.671875$$

C. **MIDDLE SUM**

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{n+3}{4} + \frac{n+4}{4}}{2} = \frac{\frac{n+3+n+4}{4}}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{2n+7}{4}}{2} = \frac{2n+7}{4} \cdot \frac{1}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{2n+7}{8}$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = f\left(\frac{2n+7}{8}\right) = \left(\frac{2n+7}{8}\right)^3$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = \left(\frac{2n+7}{8}\right)^3$$

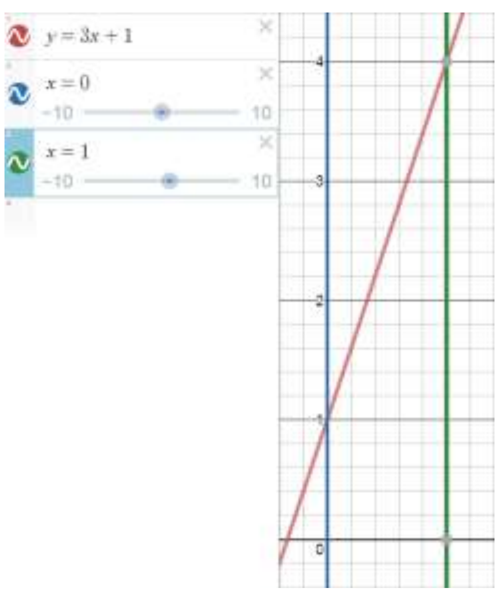
$$A_n(middle\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_n + x_{n-1}}{2}\right)$$

$$A_4(middle\ sum) = \sum_{n=1}^{4}\left(\frac{1}{4}\right) \cdot \left(\frac{2n+7}{8}\right)^3$$

$$A_4(middle\ sum) = \frac{1}{4}\sum_{n=1}^{4}\left(\frac{2n+7}{8}\right)^3 = \frac{1}{4}\left(\frac{477}{32}\right)$$

$$A_4(middle\ sum) = \frac{477}{128} = 3.7265625$$

3. $y = 3x + 1$ from $x = 0$ to $x = 1, \mathrm{n} = 4$



$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{1-0}{4}$$

$$\Delta x = \frac{1}{4}$$

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AREA UNDER A CURVE Assignment

A. **LEFT SUM**

$$x_{n-1} = a + (n-1)\cdot\Delta x = 0 + (n-1)\left(\frac{1}{4}\right)$$

$$x_{n-1} = \frac{n-1}{4}$$

$$f(x_{n-1}) = f\left(\frac{n-1}{4}\right) = 3\left(\frac{n-1}{4}\right) + 1$$

$$f(x_{n-1}) = 3\left(\frac{n-1}{4}\right) + 1$$

$$A_n(left\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f(x_{n-1})$$

$$A_4(left\ sum) = \sum_{n=1}^{4}\left(\frac{1}{4}\right)\cdot\left(3\left(\frac{n-1}{4}\right)+1\right)$$

$$A_4(left\ sum) = \frac{1}{4}\sum_{n=1}^{4}\left(3\left(\frac{n-1}{4}\right)+1\right) = \frac{1}{4}\left(\frac{17}{2}\right)$$

$$A_4(left\ sum) = \frac{17}{8} = 2.125$$

B. **RIGHT SUM**

$$x_n = a + n\cdot\Delta x = 0 + n\left(\frac{1}{4}\right)$$

$$x_n = \frac{n}{4}$$

$$f(x_n) = f\left(\frac{n}{4}\right) = 3\left(\frac{n}{4}\right) + 1$$

$$f(x_n) = 3\left(\frac{n}{4}\right) + 1$$

$$A_n(\text{right}\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f(x_n)$$

$$A_4(\text{right}\ sum) = \sum_{n=1}^{4}\left(\frac{1}{4}\right)\cdot\left(3\left(\frac{n}{4}\right)+1\right)$$

$$A_4(\text{right}\ sum) = \frac{1}{4}\sum_{n=1}^{4}\left(3\left(\frac{n}{4}\right)+1\right) = \frac{1}{4}\left(\frac{23}{2}\right)$$

$$A_4(\text{right}\ sum) = \frac{23}{8} = 2.875$$

C. **MIDDLE SUM**

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{n-1}{4} + \frac{n}{4}}{2} = \frac{\frac{n-1+n}{4}}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{2n-1}{4}}{2} = \frac{2n-1}{4}\cdot\frac{1}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{2n-1}{8}$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = f\left(\frac{2n-1}{8}\right) = 3\left(\frac{2n-1}{8}\right) + 1$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = 3\left(\frac{2n-1}{8}\right) + 1$$

$$A_n(middle\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f\left(\frac{x_n + x_{n-1}}{2}\right)$$

$$A_s(middle\ sum) = \sum_{n=1}^{4}\left(\frac{1}{4}\right)\cdot\left(3\left(\frac{2n-1}{8}\right)+1\right)$$

$$A_4(middle\ sum) = \frac{1}{4}\sum_{n=1}^{4}\left(3\left(\frac{2n-1}{8}\right)+1\right)$$

$$A_4(middle\ sum) = \frac{1}{4}(10)$$

$$A_4(middle\ sum) = \frac{10}{4} = 2.5$$

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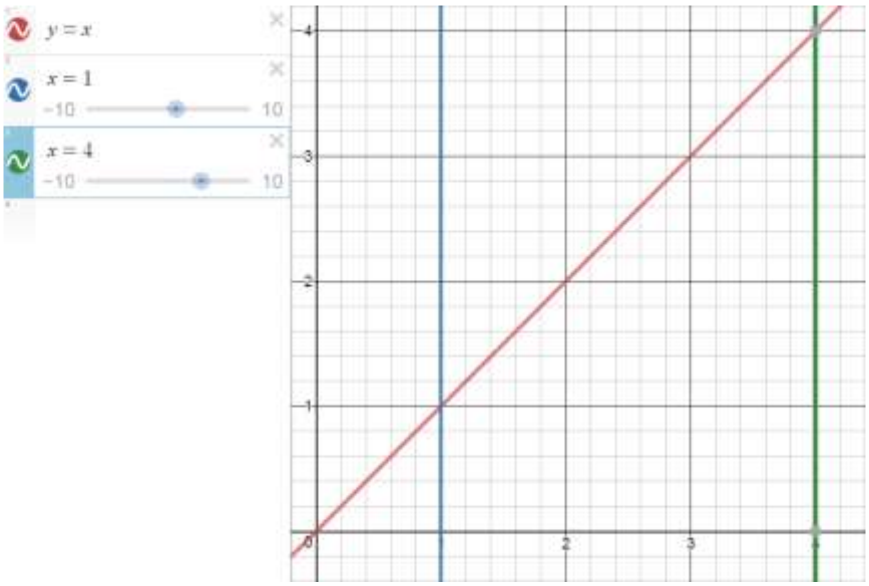
AREA UNDER A CURVE Assignment

4. $y = x$ from $x = 1$ to $x = 4, n = 6$



$$\Delta x = \frac{b-a}{n}$$
$$\Delta x = \frac{4-1}{6}$$
$$\Delta x = \frac{3}{6}$$
$$\Delta x = \frac{1}{2}$$

A. **LEFT SUM**

$$x_{n-1} = a + (n-1)\cdot \Delta x = 1 + (n-1)\left(\frac{1}{2}\right)$$
$$x_{n-1} = 1 + \frac{n-1}{2} = \frac{2}{2} + \frac{n-1}{2} = \frac{n-1+2}{2}$$
$$x_{n-1} = \frac{n+1}{2}$$
$$f(x_{n-1}) = f\left(\frac{n+1}{2}\right) = \frac{n+1}{2}$$
$$f(x_{n-1}) = \frac{n+1}{2}$$

$$A_n(left\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f(x_{n-1})$$
$$A_6(left\ sum) = \sum_{n=1}^{6}\left(\frac{1}{2}\right)\cdot\left(\frac{n+1}{2}\right)$$
$$A_6(left\ sum) = \frac{1}{2}\sum_{n=1}^{6}\left(\frac{n+1}{2}\right) = \frac{1}{2}\left(\frac{27}{2}\right)$$
$$A_6(left\ sum) = \frac{27}{4} = 6.75$$

B. **RIGHT SUM**

$$x_n = a + n\cdot \Delta x = 1 + n\left(\frac{1}{2}\right) = \frac{2}{2} + \frac{n}{2}$$
$$x_n = \frac{n+2}{2}$$
$$f(x_n) = f\left(\frac{n+2}{2}\right) = \frac{n+2}{2}$$
$$f(x_n) = \frac{n+2}{2}$$

$$A_n(\text{right}\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f(x_n)$$
$$A_6(\text{right}\ sum) = \sum_{n=1}^{6}\left(\frac{1}{2}\right)\cdot\left(\frac{n+2}{2}\right)$$
$$A_6(\text{right}\ sum) = \frac{1}{2}\sum_{n=1}^{6}\left(\frac{n+2}{2}\right) = \frac{1}{2}\left(\frac{33}{2}\right)$$
$$A_6(\text{right}\ sum) = \frac{33}{4} = 8.25$$

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AREA UNDER A CURVE Assignment

C. **MIDDLE SUM**

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{n+1}{2} + \frac{n+2}{2}}{2} = \frac{\frac{n+1+n+2}{2}}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{2n+3}{2}}{2} = \frac{2n+3}{2} \cdot \frac{1}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{2n+3}{4}$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = f\left(\frac{2n+3}{4}\right) = \frac{2n+3}{4}$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = \frac{2n+3}{4}$$

$$A_n(middle\ sum) = \sum_{n=1}^{n} \left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_n + x_{n-1}}{2}\right)$$

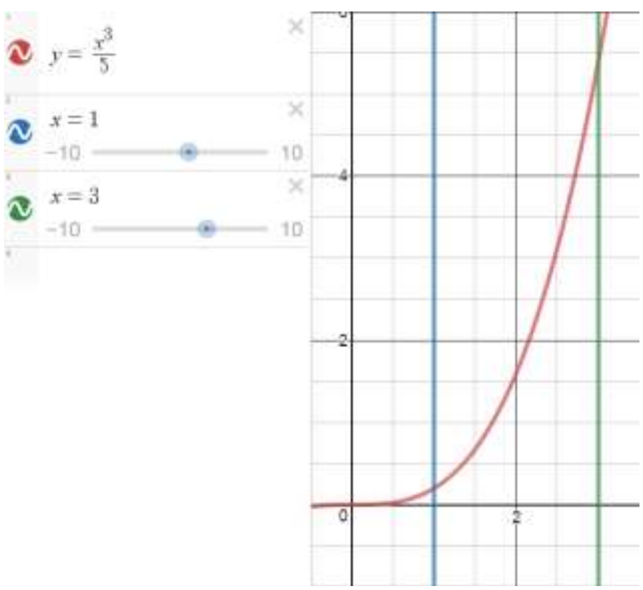
$$A_6(middle\ sum) = \sum_{n=1}^{6} \left(\frac{1}{2}\right) \cdot \left(\frac{2n+3}{4}\right)$$

$$A_6(middle\ sum) = \frac{1}{2}\sum_{n=1}^{6} \left(\frac{2n+3}{4}\right)$$

$$A_6(middle\ sum) = \frac{1}{2}(15)$$

$$A_6(middle\ sum) = \frac{15}{2} = 7.5$$

5. $y = \frac{x^3}{5} = \frac{1}{5}x^3$ from $x = 1$ to $x = 3, \mathrm{n} = 8$



$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{3-1}{8}$$

$$\Delta x = \frac{2}{8}$$

$$\Delta x = \frac{1}{4}$$

A. **LEFT SUM**

$$x_{n-1} = a + (n-1) \cdot \Delta x = 1 + (n-1)\left(\frac{1}{4}\right)$$

$$x_{n-1} = 1 + \frac{n-1}{4} = \frac{4}{4} + \frac{n-1}{4} = \frac{n-1+4}{4}$$

$$x_{n-1} = \frac{n+3}{4}$$

$$f(x_{n-1}) = f\left(\frac{n+3}{4}\right) = \frac{1}{5}\left(\frac{n+3}{4}\right)^3$$

$$f(x_{n-1}) = \frac{1}{5}\left(\frac{n+3}{4}\right)^3$$

$$A_n(left\ sum) = \sum_{n=1}^{n} \left(\frac{b-a}{n}\right) \cdot f(x_{n-1})$$

$$A_8(left\ sum) = \sum_{n=1}^{8} \left(\frac{1}{4}\right) \cdot \frac{1}{5}\left(\frac{n+3}{4}\right)^3$$

$$A_8(left\ sum) = \frac{1}{20}\sum_{n=1}^{8} \left(\frac{n+3}{4}\right)^3 = \frac{1}{20}\left(\frac{135}{2}\right)$$

$$A_8(left\ sum) = \frac{135}{40} = 3.375$$

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AREA UNDER A CURVE Assignment

B. **RIGHT SUM**

$$x_n = a + n \cdot \Delta x = 1 + n\left(\frac{1}{4}\right) = \frac{4}{4} + \frac{n}{4}$$

$$x_n = \frac{n+4}{4}$$

$$f(x_n) = f\left(\frac{n+4}{4}\right) = \frac{1}{5}\left(\frac{n+4}{4}\right)^3$$

$$f(x_n) = \frac{1}{5}\left(\frac{n+4}{4}\right)^3$$

$$A_n(\text{right } sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right) \cdot f(x_n)$$

$$A_8(\text{right } sum) = \sum_{n=1}^{8}\left(\frac{1}{4}\right) \cdot \frac{1}{5}\left(\frac{n+4}{4}\right)^3$$

$$A_8(\text{right } sum) = \frac{1}{20}\sum_{n=1}^{8}\left(\frac{n+4}{4}\right)^3 = \frac{1}{20}\left(\frac{187}{2}\right)$$

$$A_8(\text{right } sum) = \frac{187}{40} = 4.675$$

C. **MIDDLE SUM**

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{n+3}{4} + \frac{n+4}{4}}{2} = \frac{\frac{n+3+n+4}{4}}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{2n+7}{4}}{2} = \frac{2n+7}{4} \cdot \frac{1}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{2n+7}{8}$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = f\left(\frac{2n+7}{8}\right) = \frac{1}{5}\left(\frac{2n+7}{8}\right)^3$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = \frac{1}{5}\left(\frac{2n+7}{8}\right)^3$$

$$A_n(middle\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_n + x_{n-1}}{2}\right)$$

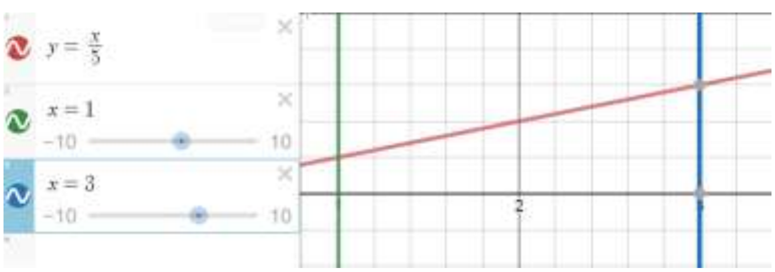
$$A_8(middle\ sum) = \sum_{n=1}^{8}\left(\frac{1}{4}\right) \cdot \frac{1}{5}\left(\frac{2n+7}{8}\right)^3$$

$$A_8(middle\ sum) = \frac{1}{20}\sum_{n=1}^{8}\left(\frac{2n+7}{8}\right)^3$$

$$A_8(middle\ sum) = \frac{1}{20}\left(\frac{319}{4}\right)$$

$$A_8(middle\ sum) = \frac{319}{80} = 3.9875$$

6. $y = \frac{x}{5}$ from $x = 1$ to $x = 3, \mathbf{n} = 4$



$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{3-1}{4}$$

$$\Delta x = \frac{2}{4}$$

$$\Delta x = \frac{1}{2}$$

A. **LEFT SUM**

$$x_{n-1} = a + (n-1) \cdot \Delta x = 1 + (n-1)\left(\frac{1}{2}\right)$$

$$x_{n-1} = 1 + \frac{n-1}{2} = \frac{2}{2} + \frac{n-1}{2} = \frac{n-1+2}{2}$$

$$x_{n-1} = \frac{n+1}{2}$$

$$f(x_{n-1}) = f\left(\frac{n+1}{2}\right) = \frac{\frac{n+1}{2}}{5} = \frac{n+1}{2}\left(\frac{1}{5}\right)$$

$$f(x_{n-1}) = \frac{n+1}{10}$$

$$A_n(left\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right) \cdot f(x_{n-1})$$

$$A_4(left\ sum) = \sum_{n=1}^{4}\left(\frac{1}{2}\right) \cdot \left(\frac{n+1}{10}\right)$$

$$A_4(left\ sum) = \frac{1}{20}\sum_{n=1}^{4}(n+1) = \frac{7}{10}$$

$$A_4(left\ sum) = 0.7$$

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AREA UNDER A CURVE Assignment

B. **RIGHT SUM**

$$x_n = a + n \cdot \Delta x = 1 + n\left(\frac{1}{2}\right) = \frac{2}{2} + \frac{n}{2}$$

$$x_n = \frac{n+2}{2}$$

$$f(x_n) = f\left(\frac{n+2}{2}\right) = \frac{\frac{n+2}{2}}{5} = \frac{n+2}{2}\left(\frac{1}{5}\right)$$

$$f(x_n) = \frac{n+2}{10}$$

$$A_n(\text{right } sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right) \cdot f(x_n)$$

$$A_4(\text{right } sum) = \sum_{n=1}^{4}\left(\frac{1}{2}\right) \cdot \left(\frac{n+2}{10}\right)$$

$$A_4(\text{right } sum) = \frac{1}{20}\sum_{n=1}^{4}(n+2) = \frac{1}{20}(18) = \frac{9}{10}$$

$$A_4(\text{right } sum) = 0.9$$

C. **MIDDLE SUM**

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{n+1}{2} + \frac{n+2}{2}}{2} = \frac{\frac{n+1+n+2}{2}}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{2n+3}{2}}{2} = \frac{2n+3}{2} \cdot \frac{1}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{2n+3}{4}$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = f\left(\frac{2n+3}{4}\right) = \frac{\frac{2n+3}{4}}{5} = \frac{2n+3}{4}\left(\frac{1}{5}\right)$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = \frac{2n+3}{20}$$

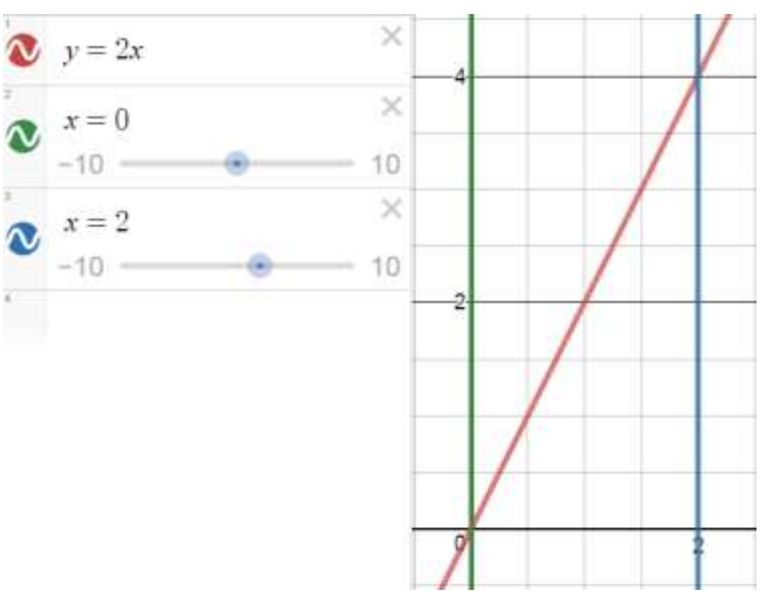
$$A_n(middle\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_n + x_{n-1}}{2}\right)$$

$$A_4(middle\ sum) = \sum_{n=1}^{4}\left(\frac{1}{2}\right) \cdot \left(\frac{2n+3}{20}\right)$$

$$A_4(middle\ sum) = \frac{1}{40}\sum_{n=1}^{4}(2n+3) = \frac{1}{40}(32) = \frac{32}{40}$$

$$A_4(middle\ sum) = 0.8$$

7. $y = 2x$ from $x = 0$ to $x = 2, n = 4$



$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{2-0}{4}$$

$$\Delta x = \frac{2}{4}$$

$$\Delta x = \frac{1}{2}$$

Name: ____ Period: __________ Date: ______________

AREA UNDER A CURVE Assignment

A. **LEFT SUM**

$$x_{n-1} = a + (n-1)\cdot \Delta x = 0 + (n-1)\left(\frac{1}{2}\right)$$

$$x_{n-1} = \frac{n-1}{2}$$

$$f(x_{n-1}) = f\left(\frac{n-1}{2}\right) = 2\left(\frac{n-1}{2}\right)$$

$$f(x_{n-1}) = n-1$$

$$A_n(left\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f(x_{n-1})$$

$$A_4(left\ sum) = \sum_{n=1}^{4}\left(\frac{1}{2}\right)\cdot (n-1)$$

$$A_4(left\ sum) = \frac{1}{2}\sum_{n=1}^{4}(n-1) = \frac{1}{2}(6) = \frac{6}{2}$$

$$A_4(left\ sum) = 3$$

B. **RIGHT SUM**

$$x_n = a + n\cdot \Delta x = 0 + n\left(\frac{1}{2}\right) = x_n = \frac{n}{2}$$

$$f(x_n) = f\left(\frac{n}{2}\right) = 2\left(\frac{n}{2}\right) = f(x_n) = n$$

$$A_n(\text{right}\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f(x_n)$$

$$A_4(\text{right}\ sum) = \sum_{n=1}^{4}\left(\frac{1}{2}\right)\cdot (n)$$

$$A_4(\text{right}\ sum) = \frac{1}{2}\sum_{n=1}^{4}(n) = \frac{1}{2}(10) = \frac{10}{2}$$

$$A_4(\text{right}\ sum) = 5$$

C. **MIDDLE SUM**

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{n-1}{2}+\frac{n}{2}}{2} = \frac{\frac{n-1+n}{2}}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{2n-1}{2}}{2} = \frac{2n-1}{2}\cdot\frac{1}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{2n-1}{4}$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = f\left(\frac{2n-1}{4}\right) = 2\left(\frac{2n-1}{4}\right)$$

$$f(x_n) = \frac{2n-1}{2}$$

$$A_n(middle\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f\left(\frac{x_n + x_{n-1}}{2}\right)$$

$$A_4(middle\ sum) = \sum_{n=1}^{4}\left(\frac{1}{2}\right)\cdot\left(\frac{2n-1}{2}\right)$$

$$A_4(middle\ sum) = \frac{1}{4}\sum_{n=1}^{4}(2n-1) = \frac{1}{4}(16) = \frac{16}{4}$$

$$A_4(middle\ sum) = 4$$