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AREA UNDER A CURVE Bell Work

Find the area between each curve and the x-axis for the given interval and subinterval using left sum, right sum, and middle sum.

1. $\boldsymbol{y = x^4}$ from $\boldsymbol{x = -1}$ to $\boldsymbol{x = 0, \mathrm{n} = 8}$

2. $\boldsymbol{y = x^2 + 6x}$ from $\boldsymbol{x = 0}$ to $\boldsymbol{x = 4, \mathrm{n} = 8}$

3. $\boldsymbol{y = x^2 - x + 1}$ from $\boldsymbol{x = 0}$ to $\boldsymbol{x = 3, \mathrm{n} = 6}$

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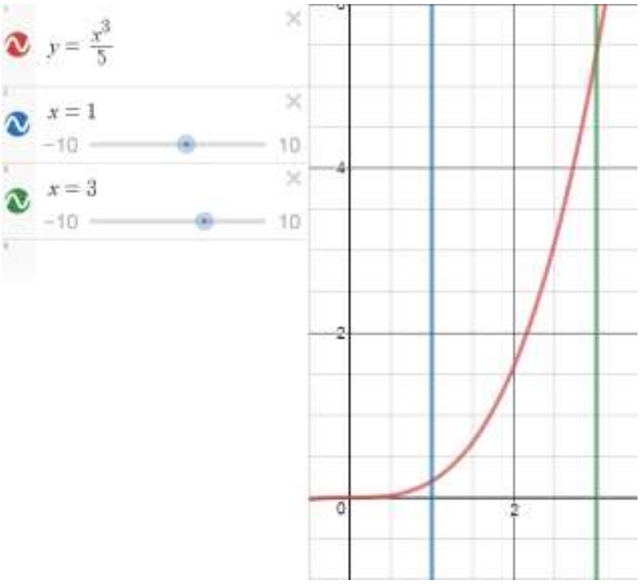
AREA UNDER A CURVE Bell Work

ANSWER

Find the area between each curve and the x-axis for the given interval and subinterval using left sum, right sum, and middle sum.

1. $y = x^4$ from $x = -1$ to $x = 0, n = 8$



$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{0-(-1)}{8}$$

$$\Delta x = \frac{1}{8}$$

$$\Delta x = \frac{1}{8}$$

A. **LEFT SUM**

$$x_{n-1} = a + (n-1)\cdot \Delta x = -1 + (n-1)\left(\frac{1}{8}\right)$$

$$x_{n-1} = -1 + \frac{n-1}{8} = -\frac{8}{8} + \frac{n-1}{8} = \frac{n-1-8}{8}$$

$$x_{n-1} = \frac{n-9}{8}$$

$$f(x_{n-1}) = f\left(\frac{n-9}{8}\right) = \left(\frac{n-9}{8}\right)^4$$

$$f(x_{n-1}) = \left(\frac{n-9}{8}\right)^4$$

$$A_n(left\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f(x_{n-1})$$

$$A_8(left\ sum) = \sum_{n=1}^{8}\left(\frac{1}{8}\right)\cdot\left(\frac{n-9}{8}\right)^4$$

$$A_8(left\ sum) = \frac{1}{8}\sum_{n=1}^{8}\left(\frac{n-9}{8}\right)^4 = \frac{1}{8}\left(\frac{2193}{1024}\right)$$

$$A_8(left\ sum) = \frac{2193}{8192} = 0.2677$$

B. **RIGHT SUM**

$$x_n = a + n\cdot \Delta x = -1 + n\left(\frac{1}{8}\right) = -\frac{8}{8} + \frac{n}{8}$$

$$x_n = \frac{n-8}{8}$$

$$f(x_n) = f\left(\frac{n-8}{8}\right) = \left(\frac{n-8}{8}\right)^4$$

$$f(x_n) = \left(\frac{n-8}{8}\right)^4$$

$$A_n(\text{right}\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f(x_n)$$

$$A_8(\text{right}\ sum) = \sum_{n=1}^{8}\left(\frac{1}{8}\right)\cdot\left(\frac{n-8}{8}\right)^4$$

$$A_8(\text{right}\ sum) = \frac{1}{8}\sum_{n=1}^{8}\left(\frac{n-8}{8}\right)^4 = \frac{1}{8}\left(\frac{1169}{1024}\right)$$

$$A_8(\text{right}\ sum) = \frac{1169}{8192} = 0.1427$$

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C. MIDDLE SUM

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{n-9}{8} + \frac{n-8}{8}}{2} = \frac{\frac{n-9+n-8}{8}}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{2n-17}{8}}{2} = \frac{2n-17}{8} \cdot \frac{1}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{2n-17}{16}$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = f\left(\frac{2n-17}{16}\right) = \left(\frac{2n-17}{16}\right)^4$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = \left(\frac{2n-17}{16}\right)^4$$

$$A_n(middle\ sum) = \sum_{n=1}^{n} \left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_n + x_{n-1}}{2}\right)$$

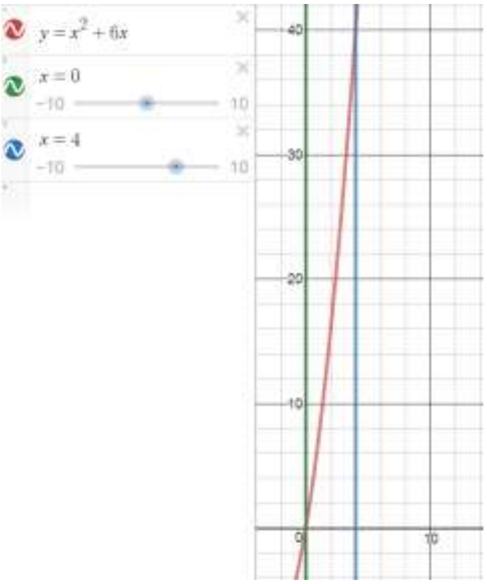
$$A_8(middle\ sum) = \sum_{n=1}^{8} \left(\frac{1}{8}\right) \cdot \left(\frac{2n-17}{16}\right)^4$$

$$A_8(middle\ sum) = \frac{1}{8} \sum_{n=1}^{8} \left(\frac{2n-17}{16}\right)^4$$

$$A_8(middle\ sum) = \frac{1}{8}\left(\frac{12937}{8192}\right)$$

$$A_8(middle\ sum) = \frac{12937}{65536} = 0.1974$$

2. $y = x^2 + 6x$ from $x = 0$ to $x = 4, n = 8$



$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{4-0}{8}$$

$$\Delta x = \frac{4}{8}$$

$$\Delta x = \frac{1}{2}$$

A. LEFT SUM

$$x_{n-1} = a + (n-1) \cdot \Delta x = 0 + (n-1)\left(\frac{1}{2}\right)$$

$$x_{n-1} = \frac{n-1}{2}$$

$$f(x_{n-1}) = f\left(\frac{n-1}{2}\right) = \left(\frac{n-1}{2}\right)^2 + 6\left(\frac{n-1}{2}\right)$$

$$f(x_{n-1}) = \left(\frac{n^2-2n+1}{4}\right) + \left(\frac{6n-6}{2}\right)\left(\frac{2}{2}\right)$$

$$f(x_{n-1}) = \left(\frac{n^2-2n+1}{4}\right) + \left(\frac{12n-12}{4}\right)$$

$$f(x_{n-1}) = \frac{n^2+10n-11}{4}$$

$$A_n(left\ sum) = \sum_{n=1}^{n} \left(\frac{b-a}{n}\right) \cdot f(x_{n-1})$$

$$A_8(left\ sum) = \sum_{n=1}^{8} \left(\frac{1}{2}\right) \cdot \left(\frac{n^2+10n-11}{4}\right)$$

$$A_8(left\ sum) = \frac{1}{8} \sum_{n=1}^{8} (n^2+10n-11) = \frac{1}{8}(476)$$

$$A_8(left\ sum) = \frac{119}{2} = 59.5$$

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B. **RIGHT SUM**

$$x_n = a + n \cdot \Delta x = 0 + n\left(\frac{1}{2}\right) = x_n = \frac{n}{2}$$

$$f(x_n) = f\left(\frac{n}{2}\right) = \left(\frac{n}{2}\right)^2 + 6\left(\frac{n}{2}\right)$$

$$f(x_n) = \frac{n^2}{4} + \frac{6n}{2} = \frac{n^2}{4} + \frac{12n}{4}$$

$$f(x_n) = \frac{n^2 + 12n}{4}$$

$$A_n(\text{right } sum) = \sum_{n=1}^{n} \left(\frac{b-a}{n}\right) \cdot f(x_n)$$

$$A_8(\text{right } sum) = \sum_{n=1}^{8} \left(\frac{1}{2}\right) \cdot \left(\frac{n^2 + 12n}{4}\right)$$

$$A_8(\text{right } sum) = \frac{1}{8}\sum_{n=1}^{8} (n^2 + 12n) = \frac{1}{8}(636)$$

$$A_8(\text{right } sum) = \frac{159}{2} = 79.5$$

C. **MIDDLE SUM**

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{n-1}{2} + \frac{n}{2}}{2} = \frac{\frac{n-1+n}{2}}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{2n-1}{2}}{2} = \frac{2n-1}{2} \cdot \frac{1}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{2n-1}{4}$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = f\left(\frac{2n-1}{4}\right)$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = \left(\frac{2n-1}{4}\right)^2 + 6\left(\frac{2n-1}{4}\right)$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = \left(\frac{4n^2 - 4n + 1}{16}\right) + \left(\frac{12n - 6}{4}\right)$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = \left(\frac{4n^2 - 4n + 1}{16}\right) + \left(\frac{48n - 24}{4}\right)$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = \frac{4n^2 + 44n - 23}{16}$$

$$A_n(middle\ sum) = \sum_{n=1}^{n} \left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_n + x_{n-1}}{2}\right)$$

$$A_8(middle\ sum) = \sum_{n=1}^{8} \left(\frac{1}{2}\right) \cdot \left(\frac{4n^2 + 44n - 23}{16}\right)$$

$$A_8(middle\ sum) = \frac{1}{32}\sum_{n=1}^{8} (4n^2 + 44n - 23)$$

$$A_8(middle\ sum) = \frac{1}{32}(2216)$$

$$A_8(middle\ sum) = \frac{277}{4} = 69.25$$

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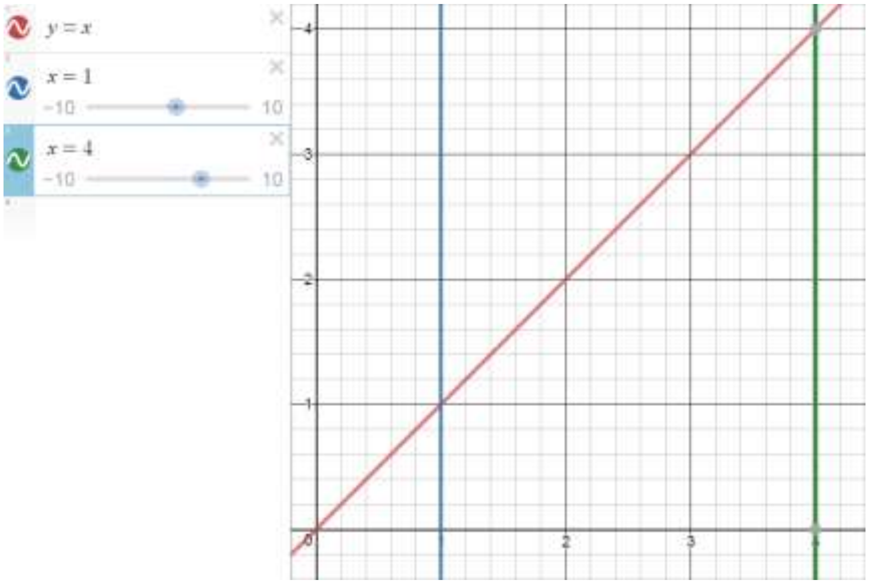
3. $y = x^2 - x + 1$ from $x = 0$ to $x = 3, \mathrm{n} = 6$



$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{3-0}{6}$$

$$\Delta x = \frac{3}{6}$$

$$\Delta x = \frac{1}{2}$$

A. **LEFT SUM**

$$x_{n-1} = a + (n-1)\cdot \Delta x = 0 + (n-1)\left(\frac{1}{2}\right)$$

$$x_{n-1} = \frac{n-1}{2}$$

$$f(x_{n-1}) = f\left(\frac{n-1}{2}\right) = \left(\frac{n-1}{2}\right)^2 - \left(\frac{n-1}{2}\right) + 1$$

$$f(x_{n-1}) = \left(\frac{n^2-2n+1}{4}\right) - \left(\frac{n-1}{2}\right)\left(\frac{2}{2}\right) + 1\left(\frac{4}{4}\right)$$

$$f(x_{n-1}) = \left(\frac{n^2-2n+1}{4}\right) + \left(\frac{-2n+2}{4}\right) + \left(\frac{4}{4}\right)$$

$$f(x_{n-1}) = \frac{n^2-4n+7}{4}$$

$$A_n(left\ sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f(x_{n-1})$$

$$A_6(left\ sum) = \sum_{n=1}^{6}\left(\frac{1}{2}\right)\cdot\left(\frac{n^2-4n+7}{4}\right)$$

$$A_6(left\ sum) = \frac{1}{8}\sum_{n=1}^{6}(n^2-4n+7) = \frac{1}{8}(49)$$

$$A_6(left\ sum) = \frac{49}{8} = 6.125$$

B. **RIGHT SUM**

$$x_n = a + n\cdot\Delta x = 0 + n\left(\frac{1}{2}\right) = \frac{n}{2}$$

$$x_n = \frac{n}{2}$$

$$f(x_n) = f\left(\frac{n}{2}\right) = \left(\frac{n}{2}\right)^2 - \left(\frac{n}{2}\right) + 1$$

$$f(x_n) = f\left(\frac{n}{2}\right) = \frac{n^2}{4} - \left(\frac{2n}{4}\right) + \frac{4}{4}$$

$$f(x_n) = \frac{n^2-2n+4}{4}$$

$$A_n(\text{right } sum) = \sum_{n=1}^{n}\left(\frac{b-a}{n}\right)\cdot f(x_n)$$

$$A_6(\text{right } sum) = \sum_{n=1}^{6}\left(\frac{1}{2}\right)\cdot\left(\frac{n^2-2n+4}{4}\right)$$

$$A_6(\text{right } sum) = \frac{1}{8}\sum_{n=1}^{6}(n^2-2n+4) = \frac{1}{8}(73)$$

$$A_6(\text{right } sum) = \frac{73}{8} = 9.125$$

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C. **MIDDLE SUM**

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{n-1}{2} + \frac{n}{2}}{2} = \frac{\frac{n-1+n}{2}}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{\frac{2n-1}{2}}{2} = \frac{2n-1}{2} \cdot \frac{1}{2}$$

$$\frac{x_n + x_{n-1}}{2} = \frac{2n-1}{4}$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = f\left(\frac{2n-1}{4}\right)$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = \left(\frac{2n-1}{4}\right)^2 - \left(\frac{2n-1}{4}\right) + 1$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = \left(\frac{4n^2 - 4n + 1}{16}\right) + \left(\frac{-8n + 4}{16}\right) + \frac{16}{16}$$

$$f\left(\frac{x_n + x_{n-1}}{2}\right) = \frac{4n^2 - 12n + 21}{16}$$

$$A_n(middle\ sum) = \sum_{n=1}^{n} \left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_n + x_{n-1}}{2}\right)$$

$$A_6(middle\ sum) = \sum_{n=1}^{6} \left(\frac{1}{2}\right) \cdot \left(\frac{4n^2 - 12n + 21}{16}\right)$$

$$A_6(middle\ sum) = \frac{1}{32} \sum_{n=1}^{6} \left(4n^2 - 12n + 21\right)$$

$$A_6(middle\ sum) = \frac{1}{32}(238)$$

$$A_6(middle\ sum) = \frac{119}{16} = 7.4375$$