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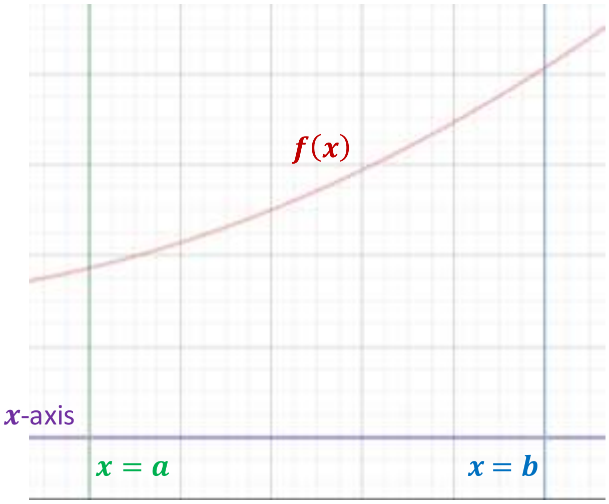
AREA UNDER A CURVE Guided Notes

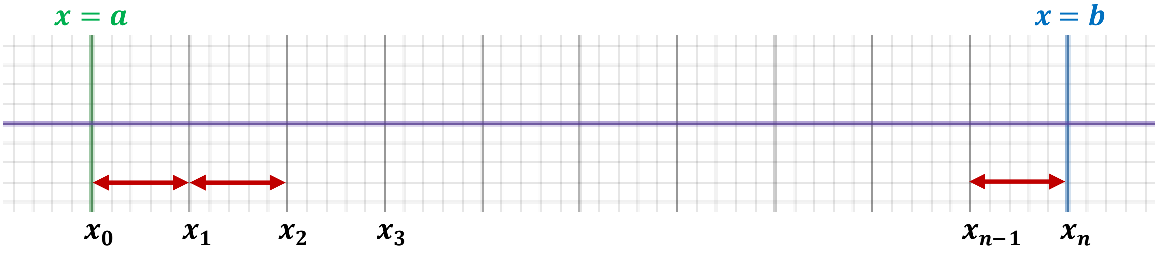
Let $f(x)$ be a continuous, non-negative function on the closed interval $[a, b]$. Determine the area bounded by $f(x)$, the vertical lines $x = a$ and $x = b$, and the x-axis.



We can use rectangles to approximate the area under the curve by subdividing or partitioning $[a, b]$ into n number of rectangles of same width. Equal width partitions are called **regular partitions**.

$$width =$$

where the partition points in terms of a and Δx:



Partition points:

$$x_0 = \quad = a = a + 0 \cdot \Delta x$$

$$x_1 = \quad a + 1 \cdot \Delta x$$

$$x_2 = \quad a + 2 \cdot \Delta x$$

$$\vdots$$

$$x_{n-1} =$$

$$x_n = \quad = b =$$

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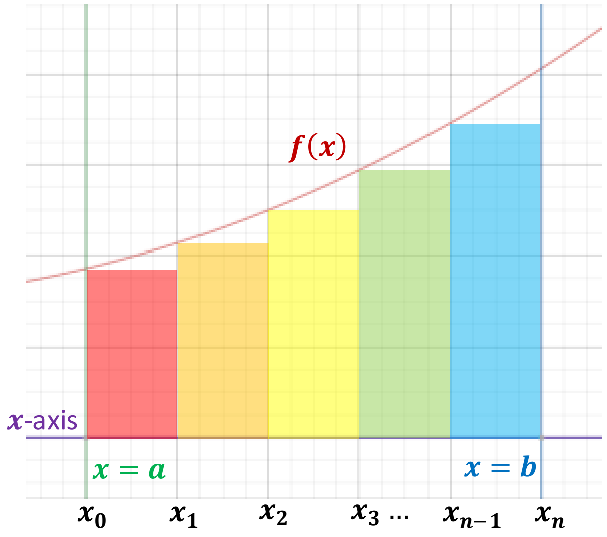
AREA UNDER A CURVE Guided Notes

If the rectangles' height are taken from **left side** of the rectangles



Partition	Height	Width (Δx)	Area
$1st$	$f(x_0)$	$x_1 - x_0$	$\Delta x \cdot f(x_0)$
$2nd$	$f(x_1)$	$x_2 - x_1$	$\Delta x \cdot f(x_1)$
$3rd$	$f(x_2)$	$x_3 - x_2$	$\Delta x \cdot f(x_2)$
…	…	…	…
$(n-1)th$	$f(x_{n-2})$	$x_{n-1} - x_{n-2}$	$\Delta x \cdot f(x_{n-2})$
nth	$f(x_{n-1})$	$x_n - x_{n-1}$	$\Delta x \cdot f(x_{n-1})$

The total A_n of the n rectangles is given by the sums of the areas

$$A_n = \left(\frac{b-a}{n}\right) \cdot f(x_0) + \left(\frac{b-a}{n}\right) \cdot f(x_2) + \left(\frac{b-a}{n}\right) \cdot f(x_3) + \cdots + \left(\frac{b-a}{n}\right) \cdot f(x_{n-2}) + \left(\frac{b-a}{n}\right) \cdot f(x_{n-1})$$

Sample Problem 1: Let $y = f(x) = 1 + \frac{1}{2}x^2$ on $[0, 2]$. Determine the left sum for regular partition into 4 subintervals.

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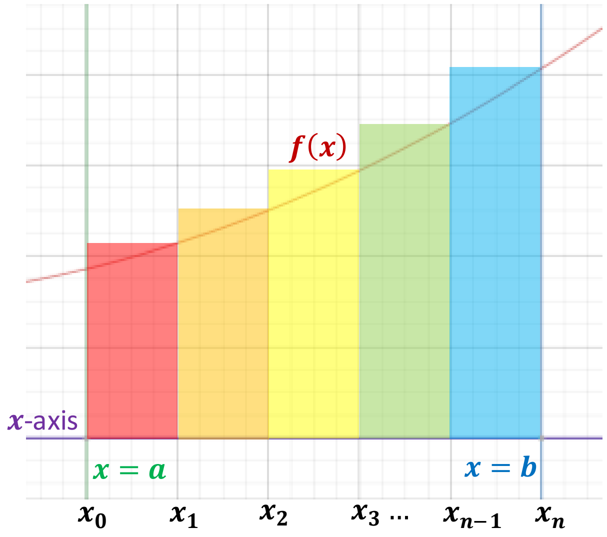
AREA UNDER A CURVE Guided Notes

If the rectangles' height are taken from the **right side** of the rectangles



Partition	**Height**	**Width (Δx)**	**Area**
$1st$	$f(x_1)$	$x_1 - x_0$	$\Delta x \cdot f(x_0)$
$2nd$	$f(x_2)$	$x_2 - x_1$	$\Delta x \cdot f(x_1)$
$3rd$	$f(x_3)$	$x_3 - x_2$	$\Delta x \cdot f(x_2)$
…	…	…	…
$(n-1)th$	$f(x_{n-1})$	$x_{n-1} - x_{n-2}$	$\Delta x \cdot f(x_{n-1})$
nth	$f(x_n)$	$x_n - x_{n-1}$	$\Delta x \cdot f(x_n)$

The total A_n of the n rectangles is given by the sums of the areas

$$A_n = \left(\frac{b-a}{n}\right) \cdot f(x_0) + \left(\frac{b-a}{n}\right) \cdot f(x_2) + \left(\frac{b-a}{n}\right) \cdot f(x_3) + \cdots + \left(\frac{b-a}{n}\right) \cdot f(x_{n-1}) + \left(\frac{b-a}{n}\right) \cdot f(x_n)$$

Sample Problem 2: Let $y = f(x) = 1 + \frac{1}{2}x^2$ on $[0, 2]$. Determine the right sum for regular partition into 4 subintervals.

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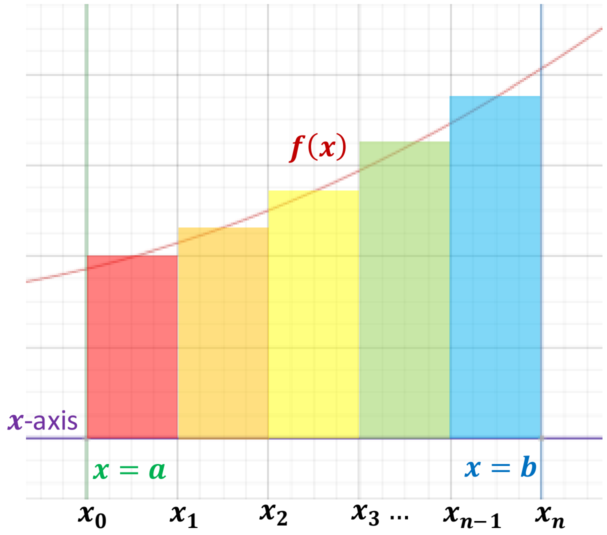
If the rectangles' height are taken from the **middle** of the rectangles



Partition	**Height**	**Width (Δx)**	**Area**
$1st$	$f\left(\frac{x_1+x_0}{2}\right)$	$x_1 - x_0$	$\Delta x \cdot f\left(\frac{x_1+x_0}{2}\right)$
$2nd$	$f\left(\frac{x_2+x_1}{2}\right)$	$x_2 - x_1$	$\Delta x \cdot f\left(\frac{x_2+x_1}{2}\right)$
$3rd$	$f\left(\frac{x_3+x_2}{2}\right)$	$x_3 - x_2$	$\Delta x \cdot f\left(\frac{x_3+x_2}{2}\right)$
…	…	…	…
$(n-1)th$	$f\left(\frac{x_{n-2}+x_{n-1}}{2}\right)$	$x_{n-1} - x_{n-2}$	$\Delta x \cdot f\left(\frac{x_{n-2}+x_{n-1}}{2}\right)$
nth	$f\left(\frac{x_n+x_{n-1}}{2}\right)$	$x_n - x_{n-1}$	$\Delta x \cdot f\left(\frac{x_n+x_{n-1}}{2}\right)$

The total A_n of the n rectangles is given by the sums of the areas

$$A_n = \left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_1+x_0}{2}\right) + \left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_2+x_1}{2}\right) + \left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_3+x_2}{2}\right) + \cdots + \left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_{n-2}+x_{n-1}}{2}\right) + \left(\frac{b-a}{n}\right) \cdot f\left(\frac{x_n+x_{n-1}}{2}\right)$$

Sample Problem 3: Let $y = f(x) = 1 + \frac{1}{2}x^2$ on $[0, 2]$. Determine the middle sum for regular partition into 4 subintervals.