SOLVING EQUATIONS

UNIT 01 LESSON 04

OBJECTIVES

STUDENTS WILL BE ABLE TO:

- Solve inequalities in one variable.
- Create Inequalities in one variable.
- Understand the difference between linear inequalities and compound linear inequalities.

KEY VOCABULARY:

- Inequality.
- Compound Inequality

An inequality says that two values are not equal.

$a \neq b$ says that a is not equal to b

There are other special symbols that show in *what way* things are not equal.

$a < b$ says that a is less than b

$a > b$ says that a is greater than b

(those two are known as strict inequality)

$a \leq b$ means that a is less than or equal to b

$a \geq b$ means that a is greater than or equal to b.

PROPERTIES OF INEQUALITIES

Anti reflexive Property	For all real numbers z, $x \nless x$ *and* $x \ngtr x$
Anti Symmetry Property	For all real numbers x and y, • If $x < y$, *then* $y \nless x$ • If $x > y$, *then* $y \ngtr x$
Transitive Property	For all real numbers x, y and z • If $x < y$, *then* $y < z$ then $x < z$ • If $x > y$, *then* $y > z$ then $x > z$

PROPERTIES OF INEQUALITIES

Addition Property	For all real numbers x,y and z, • If $x < y$ $then$ $x + z < y + z$
Subtraction Property	For all real numbers x,y and z, • If $x < y$, $then$ $x - z < y - z$

PROPERTIES OF INEQUALITIES

Multiplication Property	For all real numbers x, y and z, • If $x < y$ $then$ $\begin{cases} xz < yz \text{ } if \text{ } z > 0 \\ xz > yz \text{ } if \text{ } z < 0 \\ xz = yz, \text{ } if \text{ } z = 0 \end{cases}$ • If $x > y$ $then$ $\begin{cases} xz > yz \text{ } if \text{ } z > 0 \\ xz < yz \text{ } if \text{ } z < 0 \\ xz = yz, \text{ } if \text{ } z = 0 \end{cases}$

PROPERTIES OF INEQUALITIES

Division Property	For all real numbers x, y and z, with $z \neq 0$, • If $x < y$ *then* $\begin{cases} \frac{x}{z} < \frac{y}{z}, if\ z > 0 \\ \frac{x}{z} > \frac{y}{z}, if\ z < 0 \end{cases}$ • If $x > y$, *then* $\begin{cases} \frac{x}{z} > \frac{y}{z}, if\ z > 0 \\ \frac{x}{z} < \frac{y}{z}, if\ z < 0 \end{cases}$

Solving linear inequalities is the same as solving linear equations...

with one very important exception...

when you multiply or divide an inequality by a negative value, it changes the direction of the inequality.

A **compound inequality** is two simple inequalities joined by "and" or "or".

Solving an "And" Compound Inequality:

$3x - 9 \leq 12 \; and \; 3x - 9 \; \geq -3$

Also Written as

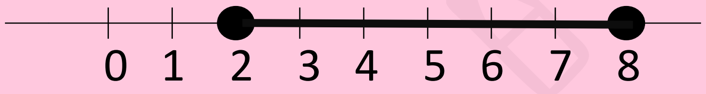
$3x - 9 \leq 12 \; \wedge \; 3x - 9 \; \geq -3$

Or Written as -3 $\leq 3x - 9 \leq 12$ 6$\leq 3x \leq 21$ 2$\leq x \leq 7$	The Common statement is sandwiched between the two inequalities. Solve as a single unit or solve each side separately.

The solution is $2 \leq x \leq 7$,

Which can be read as $x \geq 2$ ***and*** $x \leq 7$

Internal notation: [2,7]



Solving an "Or" Compound Inequality:

$2x + 3 < 7 \; or \; 5x + 5 > 25$

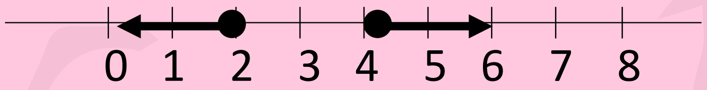
Also Written as

$[2x + 3 < 7] \vee [5x + 5 > 25]$

$2x + 3 < 7$ $2x < 4$ $x < 2$	Solve the first inequality
$5x + 5 > 25$ $5x > 20$ $x > 4$	Solve the second inequality

The solution is $x < 2$ **"Or"** $x > 4$

Interval notation: (-∞, 2)∪ (4, ∞)



PROBLEM 1

Solve the inequality

$2x - 7 > 11$

Solution

Add 7 to both sides

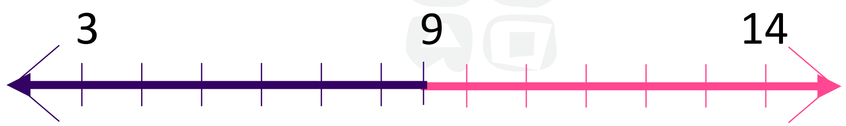
$2x - 7 + 7 > 11 + 7$

$2x > 18$

Divide both sides by 2

$x > 9$

Interval notation $(9, \infty)$



PROBLEM 2

Solve the inequality

$$2\times(3x + 9) \geq 4\times(x + 2)$$

Solution

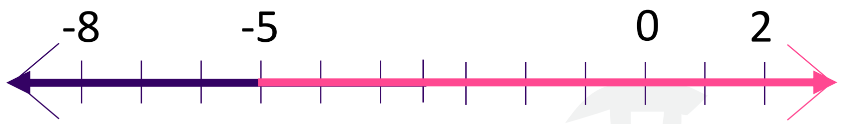
Apply the distributive property to get rid of the parentheses.

$6x + 18 \geq 4x + 8$

$6x - 4x \geq 8 - 18$

$2x \geq -10$

$x \geq -5$



Interval notation $[-5, \infty)$

PROBLEM 3

Solve the inequality

$-3\times(x + 4) \leq 2x - 2$

Solution

Apply the distributive property to get rid of the parentheses.

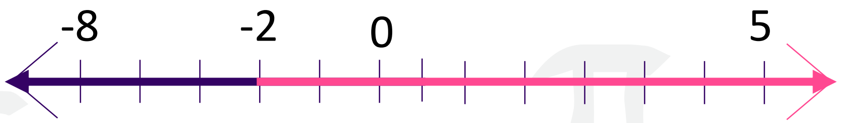
$-3x - 12 \leq 2x - 2$

$-3x - 2x \leq -2 + 12$

$-5x \leq 10$

Divide both sides by -5 and remember to change the inequality direction.

$x \geq -2$



PROBLEM 4

Solve the compound inequality

$3x - 9 \leq 12$ and $3x - 9 \geq -3$

Solution

Solve the first inequality

$3x \leq 21$

$x \leq 7$

Solve the second inequality

$3x \geq 6$

$x \geq 2$

PROBLEM 4

The solution is **$2 \leq x \leq 7$,**

which can be read **$x \geq 2$ and $x \leq 7$.**

Interval notation: **[2, 7]**