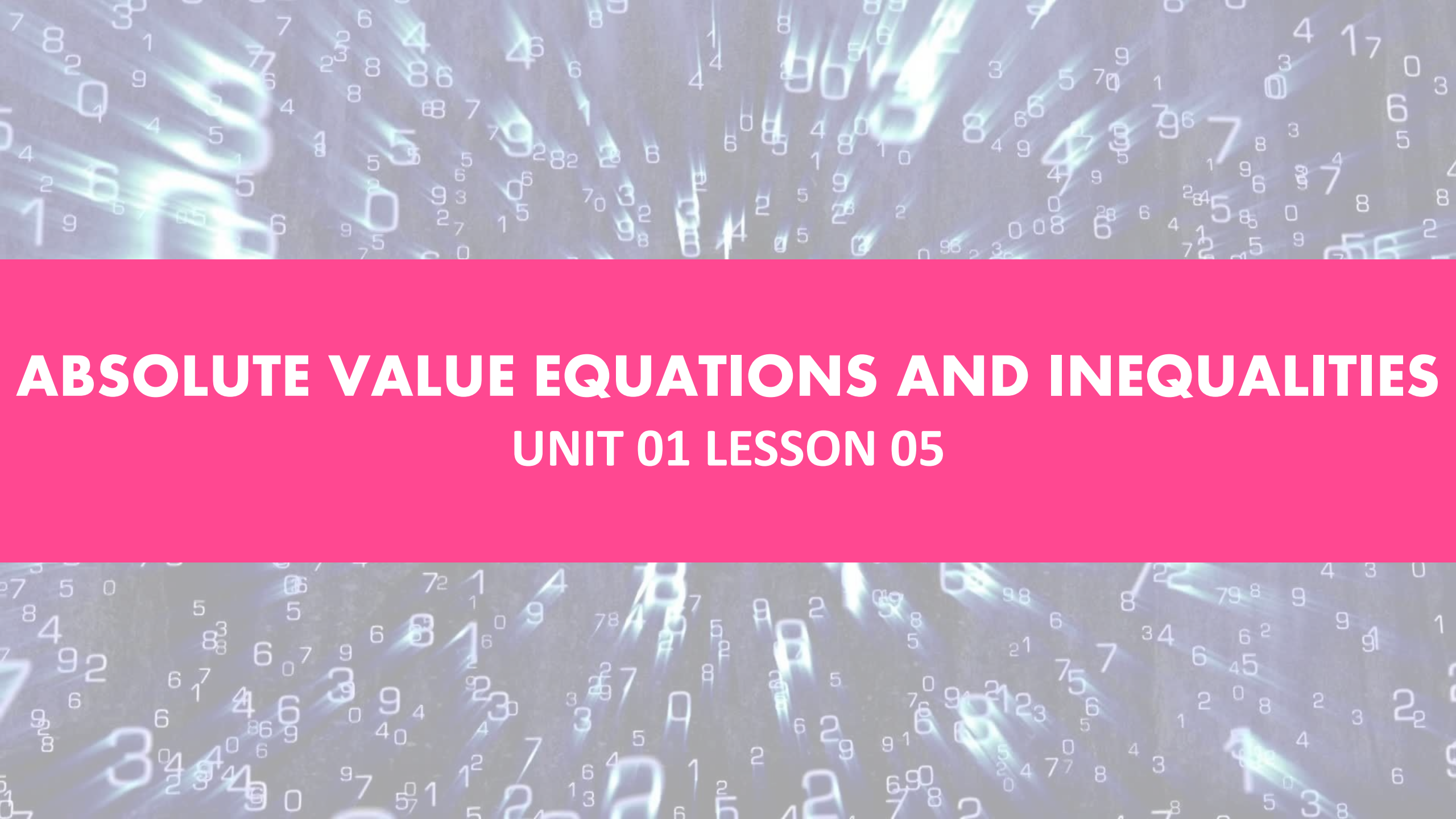


ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

UNIT 01 LESSON 05

OBJECTIVES

STUDENTS WILL BE ABLE TO:

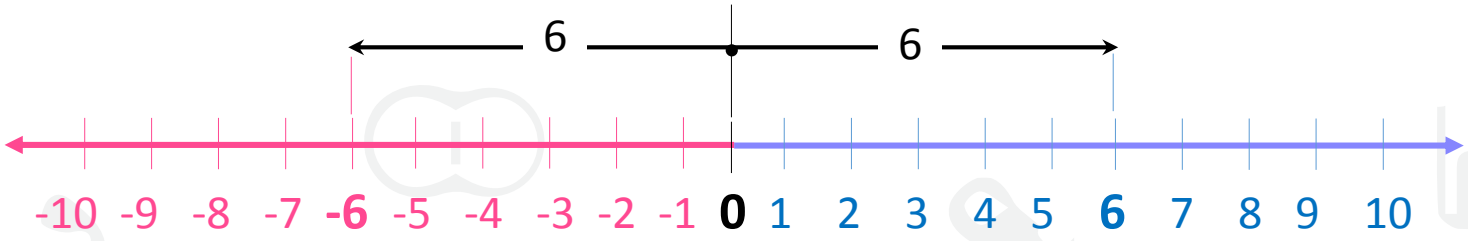
- Solve absolute value equations
- Transform an absolute value inequality into a compound inequality
- Solve absolute value inequality.

KEY VOCABULARY:

- Equation.
- Inequality.
- Absolute value.

Absolute Value means ...

... only **how far** a number is from zero:



"6" is 6 away from zero,

and "−6" is **also** 6 away from zero.

So the absolute value of 6 is **6**,

and the absolute value of −6 is also **6**

To solve an absolute value equation, isolate the absolute value on one side of the equal sign, and establish two cases:

Case 1:	**Case 2:**
$\|a\| = b$ $a = b$ Set the expression inside the absolute value symbol equal to the other given expression.	$\|a\| = b$ $a = -b$ Set the expression inside the absolute value symbol equal to the negation of the other given expression

REMEMBER!

Absolute value is always positive (or zero). An equation such as

$$|x - 3| = -5$$

is never true. It has NO solution. The answer is the empty set Ø

NOTICE!

Always **CHECK** your answers. The two cases create "derived" equations. These derived equations may not always be true equivalents to the original equation. Consequently, the roots of the derived equations MUST BE CHECKED in the original equation so that you do not list extraneous roots as answers.

PROBLEM 1

Solve |x-10| = 6

Solution

Case 1

x - 10 = 6

x = 16

Case 2

x – 10= -6

x = 4

PROBLEM 2

Solve

$2|3x-3| = 12$

Solution

First simplify the equation to get the absolute in one side.

Divide both sides by 2

$|3x - 3| = 6$

PROBLEM 2

Case 1

$3x - 3 = 6$

$3x = 9$

$x = 3$

Case 2

$3x - 3 = -6$

$3x = -6 + 3$

$3x = -3$

$x = -1$

Solving an absolute value inequality problem is similar to solving an absolute value equation.

Start by isolating the absolute value on one side of the inequality symbol, then follow the rules below:

If the symbol is $>$ $(0r \geq)$:

If $a > 0$, then the solutions to $|x| > a$ are $x > a$ or $x < -a$.

(or) if $a < 0$, all real numbers will satisfy $|x| > a$

think about it: absolute value is always positive (or zero), so , of course, it is greater than any negative number.

If the symbol is$<$(Or$\geq\leq$) :

If $a > 0$, then the solutions to $|x| < a$ and $x > -a$

Also written as : $-a < x <$ a

(and) if $a < 0$, there is no solution to $|x| < a$

think about it: absolute value is always positive (or zero), so , of course, it cannot be less than a negative number.

PROBLEM 3

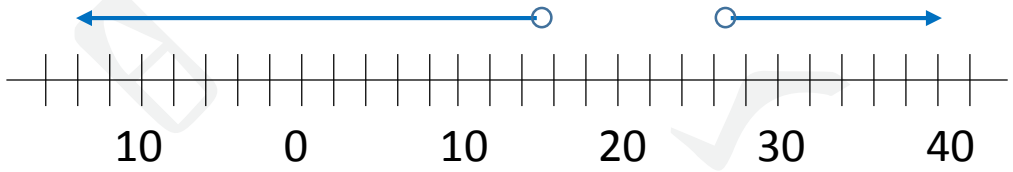
Solve $|x - 20| > 5$

Solution

Note that there are two parts to the solution and that the connecting word is "or".

Case 1	Case 2
$x - 20 > 5$	$x - 20 < -5$
$x > 25$	$x < 15$

So $x < 15$ or $x > 25$



PROBLEM 4

Solve $5 \times |x + 10| \leq 5$

Solution

We need to isolate the absolute value on one side of the equation.

$|x + 10| \leq 1$

Note that there are two parts to the solution and that the connecting word is "and".

Case 1	Case 2
$x + 10 \leq 1$	$x + 10 \geq -1$
$x \leq -9$	$x \geq -11$

So $x \geq -11$ and $x \leq -9$

Also can be written as

$-11 \leq x \leq -9$