

RATIONAL FUNCTIONS AND THEIR GRAPHS

 Guided Notes

Rational Expression is the quotient of two polynomials.

Rational function defined by a rational expression. It has an equation of the form:



where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$.

Simplest form:



where a, b, c and d are constants.

The domain of a rational function is the set of all real numbers except those real numbers that make the denominator equal to zero.

Examples:

$$f(x) = \frac{x}{x+3}$$

$$g(x) = \frac{5}{x-6}$$

$$h(x) = \frac{x+4}{(x-1)(x+4)}$$

Sample Problem 1:

Find the domain of $g(x) = \frac{x^2-7x+12}{x^2+9x+20}$.

The values that make the denominator equal to zero are restrictions on the domain of the rational function. The restrictions on the domain are described by the following definitions:

Discontinuous—a break in the line or curve on a graph

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Point of Discontinuity – the point at $x = c$ where the function is undefined (point where the denominator = 0). It is like a hole in the graph at $x = c$.

Asymptote – the line that the graph of the function approaches but never touches or crosses.

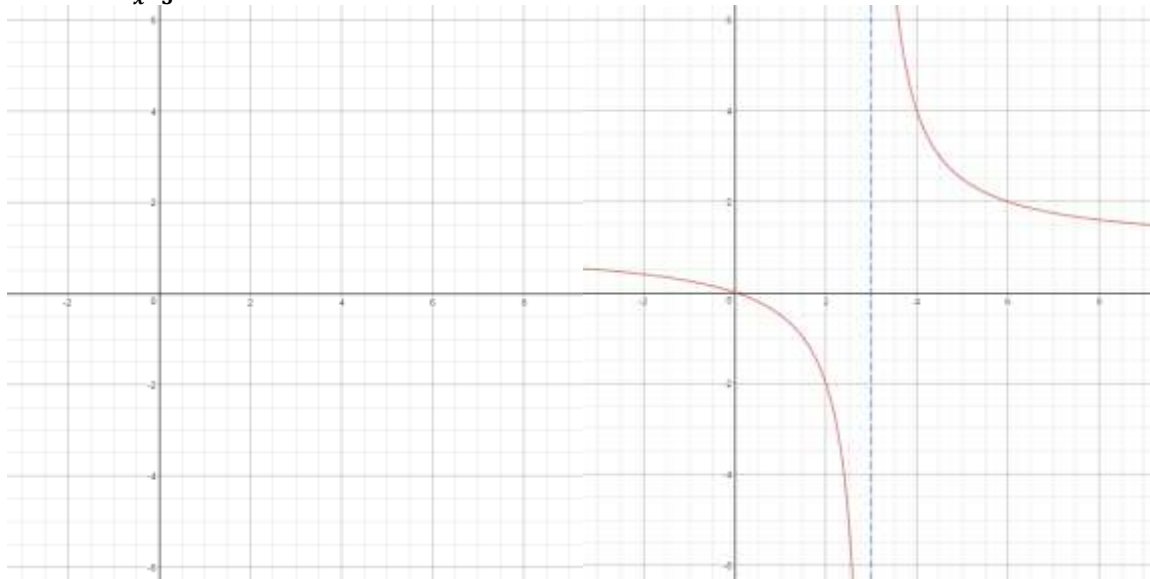
Some rational functions may not have any restrictions while others may have one or more, depending on the denominator.

Vertical Asymptote

If the rational expression of a function is written in simplest form, $f(x) = \frac{ax-b}{cx-d}$, and the function is undefined for $x = \frac{d}{c}$, then $x = \frac{d}{c}$ is a vertical asymptote.

Example:

For $f(x) = \frac{x}{x-3}$, $x = 3$ is a vertical asymptote.



Horizontal Asymptote

• $f(x) = \frac{p(x)}{q(x)} = \frac{b}{cx-d}$

If the degree of $p(x)$ is less than the degree of $q(x)$, then _____ is the equation of the horizontal asymptote of the graph of $f(x)$.

Examples:

$$f(x) = \frac{5}{x-7}$$

horizontal asymptote:

$$g(x) = \frac{3x-1}{4x^2-9}$$

horizontal asymptote:

$$h(x) = \frac{5x^2-3x}{x^3-8x^2+16x}$$

horizontal asymptote:

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$$\bullet f(x) = \frac{p(x)}{q(x)} = \frac{ax-b}{cx-d}$$

If the degree of $p(x)$ is equal to the degree of $q(x)$, and a and c are leading coefficients of $p(x)$ and $q(x)$, respectively, then _____ is the equation of the horizontal asymptote of the graph of $f(x)$.

Examples:

$$f(x) = \frac{x+2}{x-2}$$

horizontal asymptote:

$$g(x) = \frac{2x^2-5}{x^2-7x+12}$$

horizontal asymptote:

$$h(x) = \frac{6x^3+3x^2}{x^3+2x-3x}$$

horizontal asymptote:

$$\bullet f(x) = \frac{p(x)}{q(x)} = \frac{ax^2-b}{cx-d}$$

If the degree of $p(x)$ is less than the degree of $q(x)$, then the graph of $f(x)$ has _____

Examples:

$$f(x) = \frac{x+9}{5}$$

horizontal asymptote:

$$g(x) = \frac{x^2+9}{x-3}$$

horizontal asymptote:

$$h(x) = \frac{x^3-27}{2x+1}$$

horizontal asymptote:

Point of Discontinuity

If $x - b$ is a factor of both the numerator and the denominator of a rational function, then there is a point of discontinuity in the graph of the function when $x = b$, unless $x - b$ is a vertical asymptote.

Example:

$$f(x) = \frac{x^2-9}{x-3}$$

point of discontinuity:

$$g(x) = \frac{x-1}{x^2+4x-5}$$

point of discontinuity:

$$h(x) = \frac{x+3}{x^2+7x+12}$$

point of discontinuity:

Sample Problem 2:

Identify all asymptotes and holes in the graph of the rational function:

$$y = \frac{3-2x-x^2}{x^2+x-2}$$