Algebra2Coach.com

Rational Functions and their Graphs

Unit 9 Lesson 3

RATIONAL FUNCTIONS AND THEIR GRAPHS

Students will be able to:

find the roots of numbers and simplify radical expressions using roots and properties of radical expressions.

Key Vocabulary

- **Rational Expression**
- **Rational Function**
- **Vertical Asymptote**
- **Horizontal Asymptote**
- **Point of Discontinuity**

RATIONAL FUNCTIONS AND THEIR GRAPHS

Rational Expression is the quotient of two polynomials.

$$\frac{p(x)}{q(x)}$$

Rational Expression is defined by a rational expression. It has an equation in the form of:

$$f(x) = \frac{p(x)}{q(x)}, \text{ where } q(x) \neq 0.$$

Simplest form:

$$f(x) = \frac{ax - b}{cx - d}, \text{ where } a, b, \text{ and } c \text{ are constants.}$$

RATIONAL FUNCTIONS AND THEIR GRAPHS

Domain of a Rational Function

is the set of all real numbers except those real numbers that make the denominator equal to zero.

Examples:

$$f(x) = \frac{x}{x+3}$$

Domain is all real numbers except $x \neq -3$

$$g(x) = \frac{5}{x-6}$$

Domain is all real numbers except $x \neq 6$

$$h(x) = \frac{x+4}{(x-1)(x+4)}$$

Domain is all real numbers except $x \neq 1$ and $x \neq -4$

RATIONAL FUNCTIONS AND THEIR GRAPHS

Sample Problem 1:

Find the domain of $g(x) = \frac{x^2-7x+12}{x^2+9x+20}$.

Find the values of x for which the denominator, $x^2 + 9x + 20$, equals to 0.

$$x^2 + 9x + 20 = 0$$
$$(x+4)(x+5) = 0$$
$$x = -4 \text{ or } x = -5$$

The domain is all real numbers except $x = -4$ and $x = -5$.

RATIONAL FUNCTIONS AND THEIR GRAPHS

Restrictions on the domain are described by the following definitions:

Discontinuous – a break in the line or curve on a graph

Point of Discontinuity – the point at $x = c$ where the function is undefined (point where the denominator = 0). It is like a hole in the graph at $x = c$.

Asymptote – the line that the graph of the function approaches but never touches or crosses.

RATIONAL FUNCTIONS AND THEIR GRAPHS

Vertical Asymptote

If the rational expression of a function is written in simplest form, $f(x) = \frac{ax-b}{cx-d}$, and the function is undefined for $x = \frac{d}{c}$, then $x = \frac{d}{c}$ is a vertical asymptote.

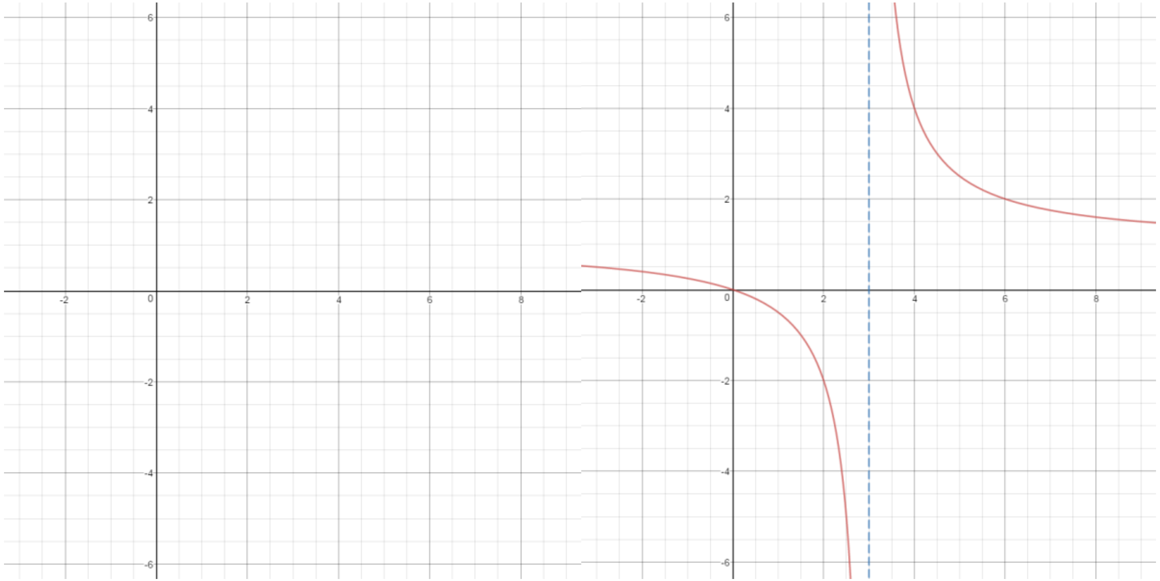
Example:

For $(x) = \frac{x}{x-3}$, $x = 3$ is a vertical asymptote.

RATIONAL FUNCTIONS AND THEIR GRAPHS

Vertical Asymptote

Graph of $f(x) = \frac{x}{x-3}$



RATIONAL FUNCTIONS AND THEIR GRAPHS

Horizontal Asymptote

If the degree of $p(x)$ is less that the degree of $q(x)$, then $y = 0$ is the equation of the horizontal asymptote of the graph of $f(x)$.

$$f(x) = \frac{p(x)}{q(x)} = \frac{b}{cx - d}$$

RATIONAL FUNCTIONS AND THEIR GRAPHS

Examples:

$$f(x) = \frac{5}{x-7}$$

horizontal asymptote:

$$y = 0$$

$$g(x) = \frac{3x-1}{4x^2-9}$$

horizontal asymptote:

$$y = 0$$

$$h(x) = \frac{5x^2-3x}{x^3-8x^2+16x}$$

horizontal asymptote:

$$y = 0$$

RATIONAL FUNCTIONS AND THEIR GRAPHS

Horizontal Asymptote

If the degree of $p(x)$ is equal to the degree of $q(x)$, and a and c are leading coefficients of $p(x)$ and $q(x)$, respectively, then

$y = \frac{a}{c}$ is the equation of the horizontal asymptote of the graph of $f(x)$.

$$f(x) = \frac{p(x)}{q(x)} = \frac{ax - b}{cx - d}$$

RATIONAL FUNCTIONS AND THEIR GRAPHS

Examples:

$$f(x) = \frac{x+2}{x-2} \qquad g(x) = \frac{2x^2-5}{x^2-7x+12} \qquad h(x) = \frac{6x^3+3x^2}{x^3+2x-3x}$$

horizontal asymptote:

$$y = \frac{a}{c} = \frac{1}{1} = y = 1$$

horizontal asymptote:

$$y = \frac{a}{c} = \frac{2}{1} = y = 2$$

horizontal asymptote:

$$y = \frac{a}{c} = \frac{6}{1} = y = 6$$

RATIONAL FUNCTIONS AND THEIR GRAPHS

Horizontal Asymptote

If the degree of $p(x)$ is less that the degree of $q(x)$, then the graph of $f(x)$ has no horizontal asymptote

$$f(x) = \frac{p(x)}{q(x)} = \frac{ax^2 - b}{cx - d}$$

Examples:

$f(x) = \frac{x+9}{5}$

No horizontal asymptote

$g(x) = \frac{x^2+9}{x-3}$

No horizontal asymptote

$h(x) = \frac{x^3-27}{2x+1}$

No horizontal asymptote

RATIONAL FUNCTIONS AND THEIR GRAPHS

Point of Discontinuity

If $\boldsymbol{x-b}$ is a factor of both the numerator and the denominator of a rational function, then there is a point of discontinuity in the graph of the function when $\boldsymbol{x=b}$, unless $\boldsymbol{x-b}$ is a vertical asymptote.

Examples:

$$f(x)=\frac{x^2-9}{x-3}$$

$$f(x)=\frac{(x-3)(x+3)}{x-3}$$

point of discontinuity:

$$x=3$$

$$g(x)=\frac{x-1}{x^2+4x-5}$$

$$g(x)=\frac{x-1}{(x-1)(x+5)}$$

point of discontinuity:

$$x=1$$

$$h(x)=\frac{x+3}{x^2+7x+12}$$

$$h(x)=\frac{x+3}{(x+3)(x+4)}$$

point of discontinuity:

$$x=-3$$

RATIONAL FUNCTIONS AND THEIR GRAPHS

Sample Problem 2:

Identify all asymptotes and holes in the graph of the rational function:

$$y = \frac{3 - 2x - x^2}{x^2 + x - 2} = \frac{-(x^2 + 2x - 3)}{x^2 + x - 2} = \frac{-(x + 3)(x - 1)}{(x - 1)(x + 2)}$$

$$y = -\frac{x + 3}{x + 2}$$

Point of Discontinuity : $x = 1$

Vertical Asymptote : $x = -2$

Horizontal Asymptote : $y = -\frac{1}{1} \Rightarrow y = -1$