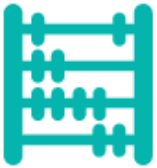

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Properties of Logarithms

Unit 8 Lesson 4

Properties of Logarithms

Students will be able to:

simplify and evaluate logarithmic expressions by applying the properties of logarithms

Key Vocabulary

- **Product property**
- **Quotient property**
- **Power property**
- **Change of Base Formula**

Properties of Logarithms

$b > 0, b \neq 1$
$a > 0, x > 0, y > 0$

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b b^k = k$
4. $b^{\log_b a} = a$

} Inverse properties

5. $\log_b xy = \log_b x + \log_b y$ — Product property
6. $\log_b \frac{x}{y} = \log_b x - \log_b y$ — Quotient property
7. $\log_b a^n = n \log_b a$ — Power property

Properties of Logarithms

Logarithmic Property	Exponential equivalent	Example
1. $log_b\ 1 = 0$	$b^0 = 1$	$log_{24}\ 1 = 0$
2. $log_b\ b = 1$	$b^1 = b$	$log_{42}\ 42 = 1$
3. $log_b\ b^k = k$	$b^k = b^k$	$log_7\ 7^2 = 2$
4. $b^{log_b\ a} = a$	$log_b\ a = log_b\ a$	$5^{log_5\ 8} = 8$

Properties of Logarithms

Problem 1

Use the properties of logarithms to evaluate expressions:

a) $\log_{\frac{1}{5}} \frac{1}{5}$

b) $\log_{0.5} 0.5^3$

c) $8^{\log_8 64}$

a) $\log_{\frac{1}{5}} \frac{1}{5} = 1$

b) $\log_{0.5} 0.5^3 = 3$

c) $8^{\log_8 64} = 64$

Properties of Logarithms

Product property

5. $\log_b xy = \log_b x + \log_b y$

$b^n b^m = b^{n+m}$

Quotient property

6. $\log_b \frac{x}{y} = \log_b x - \log_b y$

$\frac{b^n}{b^m} = b^{n+m}$

Examples:

1. $\log_6 9 + \log_6 4 = \log_6 36 = 2$
2. $\log_{\frac{1}{5}} 100 - \log_{\frac{1}{5}} 4 = \log_{\frac{1}{5}} \frac{100}{4} = \log_{\frac{1}{5}} 25 = -2$

Properties of Logarithms

Power property

7. $\log_b a^n = n \log_b a$

$(a^n)^k = a^{nk}$

Examples:

1. $\log_5 25^8 = 8 \log_5 25 = 8 \cdot 2 = 16$

2. $\log_2 0.5^{10} = 10 \log_2 \frac{1}{2} = 10 \cdot (-1) = -10$

Problem 2

Write logarithmic expression as a single logarithm:

$$\log_3 324 - 2\log_3 2$$

Solution

$$\log_3 324 - 2\log_3 2 = \log_3 324 - \log_3 2^2 = \log_3 324 - \log_3 4 =$$

$$= \log_3 \frac{324}{4} = \log_3 81 = 4$$

Answer: $\log_3 324 - 2\log_3 2 = 4$

Properties of Logarithms

Change of Base Formula

$$\log_b a = \frac{\log_k a}{\log_k b}$$

$k > 0, k \neq 1$
$a > 0, b > 0$

$$\log_b a = \frac{1}{\log_a b}$$

Example:

$$\log_6 2 \cdot \log_2 36 = \frac{1}{\log_6 2} \cdot \log_2 36 = \frac{\log_2 36}{\log_2 6} = \log_6 36 = 2$$

Problem 3

State the property used to rewrite the expression:

1. $\log_2 12 - \log_2 3 = \log_2 4$
2. $\log_3 3x = \log_3 3 + \log_3 x$
3. $\frac{2}{3}\log_4 8 = \log_4 \sqrt[3]{64}$

1. Quotient property 2. Product property 3. Power property